【Articles】

Differentiated mixed duopoly, *ex ante* cost-reducing investment, and privatization

差別化混合複占，事前の費用削減投資，及び民営化

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(**Abstract**)

Mixed oligopoly models are among the approaches to investigate the effects of privatization. In standard models of mixed duopoly, the public firm produces more than the private firm in order to maximize the social welfare. It decreases production after privatization in order to maximize its profit, and the social welfare is reduced. They are, however, not necessarily the case in reality. Privatization today takes various forms including Public-Private Partnerships (PPPs). The theory of mixed oligopoly, however, does not fully reflect it. In the present paper we introduce *ex ante* investments for cost reduction of ancillary services and the choice of bundling or unbundling of tasks into the model of differentiated mixed duopoly where the goods produced by two firms are substitutes. We consider three regimes; government management, outsourcing, and privatization. Then we show that if the investment cost is low enough, the private firm produces more than the public firm in mixed duopoly, and the privatized firm produces more than it did before privatization. If, in addition, the substitutability of goods is also low enough, the social welfare is higher under privatization than under government manage-

ment and outsourcing. Outsourcing is the most desirable in other cases.

1. Introduction

Privatization has been a major issue in economic policy, as well as in theoretical studies. Mixed oligopoly is among the approaches to investigate this issue. Mixed oligopoly is a market where a small number of firms, public and private, compete with each other. Many researchers have used the models of mixed oligopoly to analyze the effects of privatization. Despite the global trend of privatization in the 1980s many public firms survived and are competing with private firms in various oligopolistic markets. Thus, there are discussions about what the government should do with those firms.

In standard theoretical models, private firms maximize profits while a public firm acts in order to maximize the social welfare. If it is privatized, it acts as a private firm and hence affects the social welfare. In their seminal paper De Fraja and Delbono (1989) showed that under certain conditions the welfare can be higher when the public firm maximizes its profit than when it aims to maximize social welfare, and hence the public firm should be privatized. While they considered the choice between full nationalization and full privatization, Matsumura (1998) introduced partial privatization into the model of mixed duopoly where a partially privatized public firm, which is jointly owned by the government and the private party, compete with a private firm in the market. He showed that under plausible conditions neither full nationalization nor full privatization is desirable but partial privatization is optimal. Partial privatization has been used in various studies of mixed oligopoly. For exsample, Matsumura and Kanda (2005) allowed free entry of private firms. Fujiwara (2007) introduced product differentiation into the model. Han and Ogawa (2008) considered an international mixed oligopoly and investigated the impact of market integration. Oshima (2018) considered a differentiated mixed duopoly in a two-city model.

There are, however, questions about the models of mixed oligopoly. In those models, a public firm produces more than a private firm in order to maximize the social welfare. It reduces production when it is privatized in order to maximize profit, and privatization lowers the social welfare if the firms' marginal cost is not increasing much or the number of firms is small. In reality, however, many privatized firms including British Airways and Japan Railways increased their sales after privatization. Although these firms have had to downsize unprofitable divisions, they have also improved the quality of their services and created jobs through business expansion, which have helped enhance the social welfare.

In addition, the term privatization recently refers to various forms of Public-Private Partnerships (PPPs) as well, such as concession-based PPP, Private Finance Initiative (PFI), and others. In the case of concession-based PPP, unlike the partial privatization mentioned above, there is no transfer of ownership. Instead, the government sells the right to operate the

facility to a private firm or a consortium. As a more moderate approach, the government may delegate only the operation of ancillary services to a private party. The concessionaire can make investments on behalf of the government, which affects the result of the operation of the facility. Therefore, who makes investments matters. In relation to this issue, there are discussions in contracting and bundling/unbundling literature. They include Hart (2003), Bennett and Iossa (2006), Chen and Chiu (2010), and Iossa and Martimort (2015), for example, and their conclusions vary as to whether bundling or unbundling of tasks is more desirable. Hart (2003) defined PPP as a project where two tasks, facility construction and service provision, are bundled. That is, the government contracts with a private party to build and run the facility. On the other hand, under conventional provision, the government contracts with a builder to build the facility, and then later on with another private party to run it. That is, the two tasks are unbundled. In his model the builder can make investments that affect the operation of the facility. Then he showed that PPP is desirable if the quality of the service can be well specified in the initial contract, whereas the quality of the building cannot be.

In the present paper, we consider two tasks performed at the same time, the core business and the ancillary services, for a project. Either the public firm, the contractor of ancillary services, or the PPP concessionaire can make the investment for ancillary services. Ancillary services are important for the profitability of various projects because successful privatized firms have greatly increased sales and profits from ancillary services after privatization. This suggests that the public sector does not make good use of ancillary services.

The rest of the paper is organized as follows. In section 2 we set up a model of differentiated mixed duopoly and derive the equilibria of the three regimes; government management, outsourcing, and privatization. The two tasks are bundled only under privatization. In section 3 we compare the results of the three regimes. We show that the private firm produces more than the public firm and the privatized firm produces more than before it is privatized if the investment cost is low enough, and that the public firm should be privatized if both the substitutability of the two goods and the investment cost are low enough. We also investigate the results where there is no technology for the cost-reducing investments. Section 4 concludes.

2. The model

Suppose that there are two identical facilities, facilities 1 and 2. The public firm owns facility 1 and a private firm owns facility 2. They conduct two tasks; the core business which produces and sells the goods, and the ancillary services which bear no immediate relationship to producing the goods (e.g., restaurants and shops in an airport).

As mentioned in Section 1, the term privatization has various meanings. In the present paper, privatization refers to introducing the concession-based PPP, where a private consortium operates the whole facility and is delegated from the government the right to make investment it thinks necessary. That is, the tasks of core business and ancillary services are bundled. If the public firm remains state-run, it subcontracts ancillary services in the facility

to private contractors because, say, the government officials are not familiar with the business of ancillary services. In this case the tasks of core business and ancillary services are unbundled. The public firm does not delegate the right to make investment to those contractors because (it may think that) such investments by the contractors may alter the facility and affect the core business (Section 2.1). If there is no such problem, the government can outsource ancillary services to a single contractor in a lump sum and delegate the right to make investment (Section 2.2). If the two tasks are bundled, the operator of the facility should be able to handle such problems. Then the government can leave the management of facility 1 to a private consortium and delegate the right to make investment (Section 2.3).

Consider a three-stage game with two firms, firms 1 and 2, producing differentiated goods. Firm 1 is the public firm owned by the government and firm 2 is the private firm, and hence the market is a differentiated mixed duopoly. In the first stage, the government chooses one of three options; government management of the public firm (hereafter government management), lump-sum outsourcing of the ancillary services (hereafter outsourcing), and privatization. Government management is the conventional approach where the public firm is in charge of the core business and leaves ancillary services to contractors. Outsourcing is an improved form of government management in which ancillary services are outsourced in a lump sum to a single contractor. Under privatization the management of facility 1 as a whole is left to a private consortium. In the second stage, each firm (or contractor) determines how much to make cost-reducing investment for ancillary services. In the third stage, both firms produce and sell goods. We solve the game by backward induction.

Assume that the representative consumer's utility function is described as below:

$$u(x_1, x_2) = x_1 + x_2 - \frac{x_1^2 + 2n\ x_1 x_2 + x_2^2}{2} + z, \tag{1}$$

where x_i, $i = 1,2$, is the amount of goods produced by firm i, z the amount of the numeraire good, and the parameter σ, $\sigma \in [0,1)$, the substitutability of the differentiated goods.[1)] Let p_i denote the price of good i. Then the adverse demand functions are described as follows:

$$p_i = 1 - x_i - \sigma x_j, \qquad i, j = 1,2, \quad i \neq j. \tag{2}$$

Therefore, the consumer surplus, CS, is,

$$CS = u - p_1 x_1 - p_2 x_2 - z = \frac{x_1^2 + 2\sigma x_1 x_2 + x_2^2}{2}. \tag{3}$$

The two firms face the same constant marginal cost c, $c \in (0,1)$, for producing goods. Then the profits of the core business are $(p_i - c)\ x_i$, $i = 1,2$. Assume, on the other hand, that facility i's profit of ancillary services are described as sx_i, where the parameter s is the net marginal profit. That is, the profit of ancillary services is proportional to the consumption of the core business. This is a reasonable assumption because, for example, sales of airport shops, and therefore profits, are expected to increase as the number of airport users increases. The firms

can make investments for cost reduction of the ancillary services at the second stage. It reduces the marginal cost by v_i, which are given at the third stage. Therefore, the profits of ancillary services are $(s + v_i)\ x_i$, $i = 1,2$. Assuming that the competition among contractors is fierce, we set $s = 0$ below.

2.1 Government management

Suppose that the government does not privatize the public firm. We assume that the public firm does not (or can not) take into account the profits of ancillary services of the two facilities. This corresponds to the inability of real public firms to make good use of ancillary services, as mentioned in Section 1. Then the profits of the two firms net of investment costs are described as follows:

$$\pi_1 = \left(p_1 - c\right)x_1 = \left(1 - c - x_1 - \sigma x_2\right)x_1, \tag{4}$$

$$\pi_2 = \left(p_2 - c + v_2\right)x_2 = \left(1 - c + v_2 - x_2 - \sigma x_1\right)x_2, \tag{5}$$

and the objective function that the public firm maximizes, *GO*, is as below:

$$GO = CS + \pi_1 + \left(p_2 - c\right)x_2 = \left(1 - c - \frac{x_1}{2} - \sigma x_2\right)x_1 + \left(1 - c - \frac{x_2}{2}\right)x_2. \tag{6}$$

Note that π_1 and π_2 are asymmetric under goverment management, and that the public firm does not care about π_2 but $(p_2 - c)\ x_2$. Unlike the public firm's objective function *GO*, the social welfare which is explained later includes the profits of ancillary services of the two facilities.

At the third stage, the public firm maximizes its objective function while the private firm maximizes its profit. From the first-order conditions for (6) and (5), respectively, we have,

$$\frac{\partial GO}{\partial x_1} = 1 - c - x_1 - \sigma x_2 = 0, \tag{7}$$

$$\frac{\partial \pi_2}{\partial x_2} = 1 - c + v_2 - 2x_2 - \sigma x_1 = 0. \tag{8}$$

Solving (7) and (8) for x_1 and x_2 yields,

$$x_1 = \frac{\left(1 - c\right)\left(2 - \sigma\right) - \sigma v_2}{2 - \sigma^2}, \quad x_2 = \frac{\left(1 - c\right)\left(1 - \sigma\right) + v_2}{2 - \sigma^2}. \tag{9}$$

Substituting (9) into (4) and (5) we have,

$$\pi_1 = 0, \tag{10}$$

$$\pi_2 = \frac{\left[\left(1-c\right)\left(1-\sigma\right)+v_2\right]^2}{\left(2-\sigma^2\right)^2}. \tag{11}$$

At the second stage, the two firms choose the levels of cost reduction, v_i. We assume that the cost of investment which reduces the marginal cost for ancillary services by v_i is $\delta v_i^2/2$. The cost of investment increases with the parameter δ, $\delta > 0$. Therefore, the social welfare, *SW*, which is the sum of the consumer surplus and firms' profits minus investment costs, is described as below:

$$SW = CS + \pi_1 + v_1 x_1 + \pi_2 - \frac{\delta}{2}\left(v_1^2 + v_2^2\right). \tag{12}$$

Given (4) and (5) it is clear that the profits from ancillary services of the two firms are included in the social welfare in (12). On the other hand, the overall objective function for the public firm, *OGO* is given by $OGO \equiv GO - \delta v_1^2/2$. The overall profit of the private firm, Π_2, is given by $\Pi_2 \equiv \pi_2 - \delta v_2^2/2$.[2] From the first order conditions we have,

$$\frac{\partial OGO}{\partial v_1} = -\delta v_1 = 0, \tag{13}$$

$$\frac{\partial \Pi_2}{\partial v_2} = \frac{2\left(1-c\right)\left(1-\sigma\right)+\left[2-\delta\left(2-\sigma^2\right)^2\right]v_2}{\left(2-\sigma^2\right)^2} = 0. \tag{14}$$

Hence, solving (13) and (14) we have the levels of cost reduction under government management, v_1^G and v_2^G, as follows:

$$v_1^G = 0, \quad v_2^G = \frac{2\left(1-c\right)\left(1-\sigma\right)}{-2+\delta\left(2-\sigma^2\right)^2}. \tag{15}$$

In order for v_2^G to be positive the denominator needs to be positive, and hence the following condition is necessary:

$$\delta > \frac{2}{\left(2-\sigma^2\right)^2}, \tag{16}$$

which we assume in what follows.[3] Substituting (15) into (9) we have quantities produced, x_1^G and x_2^G, as follows:

$$x_1^G = \frac{\left(1-c\right)\left[-2+\delta\left(2-\sigma\right)\left(2-\sigma^2\right)\right]}{-2+\delta\left(2-\sigma^2\right)^2}, \quad x_2^G = \frac{\delta\left(1-c\right)\left(1-\sigma\right)\left(2-\sigma^2\right)}{-2+\delta\left(2-\sigma^2\right)^2}. \tag{17}$$

Substituting (2), (3), (10), (11), (15) and (17) into (12) we have the social welfare, SW^G, as below:

$$SW^G = \frac{\left(1-c\right)^2\left[4-4\delta\left(5-2\sigma-3\sigma^2+\sigma^4\right)+\delta^2\left(2-\sigma^2\right)^2\left(7-6\sigma-2\sigma^2+2\sigma^3\right)\right]}{2\left[2-\delta\left(2-\sigma^2\right)^2\right]^2}. \tag{18}$$

Suppose that there is no such technology that can reduce the marginal cost of ancillary services. In that case, the market is an ordinary mixed duopoly and the firms do not make cost-reducing investments. Substituting $v_1 = 0$ and $v_2 = 0$ into (9) yields the amounts produced, x_1^{G0} and x_2^{G0}, as follows:

$$x_1^{G0} = \frac{\left(1-c\right)\left(2-\sigma\right)}{2-\sigma^2},\quad x_2^{G0} = \frac{\left(1-c\right)\left(1-\sigma\right)}{2-\sigma^2}. \tag{19}$$

Then we have the social welfare, SW^{G0}, as below:

$$SW^{G0} = \frac{\left(1-c\right)^2\left(7-6\sigma-2\sigma^2+2\sigma^3\right)}{2\left(2-\sigma^2\right)^2}. \tag{20}$$

Note that (19) and (20) do not include δ and depend only on c and σ.

2.2 Outsourcing

Next let us assume that the government can outsource ancillary services in a lump sum to a single contractor and delegate the right to make investment to the contractor. In this case the public firm is still in charge of the core business, and hence the third stage is the same as in Section 2.1. At the second stage, let ϕ_1 denote the profit of ancillary services of facility 1, which is given by $\phi_1 = v_1 x_1 - \delta v_1^2/2$. From the first-order condition we have,

$$\frac{\partial \phi_1}{\partial v_1} = \frac{\left(1-c\right)\left(2-\sigma\right)+\sigma v_2}{2-\sigma^2} - \delta v_1 = 0. \tag{21}$$

Firm 2's first-order condition is the same as (14). Then, solving (14) and (21) for v_1 and v_2 we obtain the levels of cost reduction under outsourcing, v_1^O and v_2^O, as follows:

$$v_1^O = \frac{\left(1-c\right)\left[-2+\delta\left(2-\sigma\right)\left(2-\sigma^2\right)\right]}{\delta\left[-2+\delta\left(2-\sigma^2\right)^2\right]},\quad v_2^O = \frac{2\left(1-c\right)\left(1-\sigma\right)}{-2+\delta\left(2-\sigma^2\right)^2}. \tag{22}$$

One can see from (14) that v_2 does not depend on v_1. Hence, v_2^O is equal to v_2^G. Substituting (22) into (9) we have the amounts produced, x_1^O and x_2^O, as below:

$$x_1^O = \frac{(1-c)\left[-2+\delta\left(2-\sigma\right)\left(2-\sigma^2\right)\right]}{-2+\delta\left(2-\sigma^2\right)^2},\quad x_2^O = \frac{\delta(1-c)(1-\sigma)\left(2-\sigma^2\right)}{-2+\delta\left(2-\sigma^2\right)^2}. \tag{23}$$

Although v_1^O is not equal to zero, one can see from (9) – (11) that neither x_1, x_2, π_1 nor π_2 depends on v_1. Therefore we have $x_1^O = x_1^G$ and $x_2^O = x_2^G$. The difference from government management is that v_1 is positive, and hence, facility 1's overall profit, Π_1, and the social welfare are larger. Substituting (10), (11), (22), and (23) into (12) we have the social welfare, SW^O, as follows:

$$SW^O = \frac{(1-c)^2\,\Lambda}{2\delta\left[2-\delta\left(2-\sigma^2\right)^2\right]^2}, \tag{24}$$

where,

$$\Lambda \equiv 4-4\delta(1-\sigma)\left(3+\sigma-\sigma^2\right)-\delta^2\left(4+8\sigma-16\sigma^3+4\sigma^4+4\sigma^5-\sigma^6\right)$$
$$+\delta^3\left(2-\sigma^2\right)^2\left(7-6\sigma-2\sigma^2+2\sigma^3\right).$$

If there is no technology to reduce the marginal cost of ancillary services, the equilibrium is the same as that of government management. This is straightforward because the only difference from government management is the *ex ante* investment. Therefore, the social welfare, SW^{O0}, is equal to SW^{G0}.

2.3 Privatization

Now let us consider the case where a concession-based PPP is introduced. Firm 1 acts as a private firm and its profit is now described as below:

$$\pi_1 = \left(p_1 - c + v_1\right)x_1 = \left(1-c+v_1-x_1-\sigma x_2\right)x_1. \tag{25}$$

At the third stage, the two firms maximize their profits (25) and (5). From the first-order conditions we have,

$$\frac{\partial \pi_i}{\partial x_i} = 1-c+v_i-2x_i-\sigma x_j = 0,\ \ i,j=1,2,\ \ i\neq j. \tag{26}$$

Solving (26) for x_1 and x_2 yields,

$$x_i = \frac{(1-c)\left(2-\sigma\right)+2v_i-\sigma v_j}{\left(2+\sigma\right)\left(2-\sigma\right)},\ \ i,j=1,2,\ \ i\neq j. \tag{27}$$

Substituting (27) back into (25) and (5) we have,

$$\pi_i = \frac{\left[\left(1-c\right)\left(2-\sigma\right)+2v_i-\sigma v_j\right]^2}{\left(2+\sigma\right)^2\left(2-\sigma\right)^2},\ \ i,j=1,2,\ \ i\neq j. \tag{28}$$

At the second stage, the two firms choose the levels of cost reduction for ancillary services. The overall profit of firm 1, Π_1, is now given by $\Pi_1 = \pi_1 - \delta v_1^2/2$, and the social welfare is described as below:

$$SW = CS + \Pi_1 + \Pi_2 . \tag{29}$$

As for firms, from the first-order conditions we have,

$$\frac{\partial \Pi_i}{\partial v_i} = \frac{4\left(1-c\right)\left(2-\sigma\right)+\left[8-\delta\left(2+\sigma\right)^2\left(2-\sigma\right)^2\right]v_i-4\sigma v_j}{\left(2+\sigma\right)^2\left(2-\sigma\right)^2}=0,\ \ i,j=1,2,\ \ i\neq j. \tag{30}$$

Solving (30), the levels of cost reduction in this pure duopoly, v_1^P and v_2^P, are as follows:

$$v_i^P = \frac{4\left(1-c\right)}{-4+\delta\left(2-\sigma\right)\left(2+\sigma\right)^2},\ \ i=1,2. \tag{31}$$

In order for v_1^P and v_2^P to be positive the following condition is necessary:

$$\delta > \frac{4}{\left(2-\sigma\right)\left(2+\sigma\right)^2}, \tag{32}$$

which we assume in what follows. Substituting (31) into (27) we have the amounts produced, x_1^P and x_2^P, as follows:

$$x_i^P = \frac{\delta\left(1-c\right)\left(2-\sigma\right)\left(2+\sigma\right)}{-4+\delta\left(2-\sigma\right)\left(2+\sigma\right)^2},\ \ i=1,2. \tag{33}$$

Therefore, substituting (31) and (33) into (29) yields the social welfare, SW^P, as follows:

$$SW^P = \frac{\delta\left(1-c\right)^2\left[-16+\delta\left(2-\sigma\right)^2\left(2+\sigma\right)^2\left(3+\sigma\right)\right]}{\left[4-\delta\left(2-\sigma\right)\left(2+\sigma\right)^2\right]^2}. \tag{34}$$

Suppose again that there is no technology for cost-reducing investment for ancillary services. That is, the market is an ordinary pure duopoly. Substituting $v_1 = 0$ and $v_2 = 0$ into (27) yields the amounts produced, x_1^{P0} and x_2^{P0}, as follows:

$$x_i^{P0} = \frac{1-c}{2+\sigma},\ \ i=1,2. \tag{35}$$

Then we have the social welfare, SW^{P0}, as below:

$$SW^{P0} = \frac{(1-c)^2(3+\sigma)}{(2+\sigma)^2}. \tag{36}$$

3. Comparison

In this section we compare the amounts produced and the social welfares under the three regimes, government management, outsourcing, and privatization.

3.1 No investments

Let us first consider the case where there is no technology that allows firms or contractors to make the cost-reducing investment. From (19) the difference in production by the public and private firms under government management is as follows:

$$x_1^{G0} - x_2^{G0} = \frac{1-c}{2-\sigma^2} > 0. \tag{37}$$

That is, the public firm produces more than the private firm. This result also holds in the case of outsourcing.

From (19) and (35), the difference in production by firm 1 before and after privatization is as below:

$$x_1^{P0} - x_1^{G0} = -\frac{2(1-c)}{(2+\sigma)(2-\sigma^2)} < 0. \tag{38}$$

Hence, the privatized firm produces less than it did before privatization. Similarly, the difference in production by firm 2 before and after privatization of firm 1 is as follows:

$$x_2^{P0} - x_2^{G0} = \frac{(1-c)\sigma}{(2+\sigma)(2-\sigma^2)} > 0. \tag{39}$$

That is, firm 2 produces more than it did before privatization of firm 1. The difference in total production before and after privatization of firm 1 is as below:

$$\left(x_1^{P0} + x_2^{P0}\right) - \left(x_1^{G0} + x_2^{G0}\right) = -\frac{(1-c)(2-\sigma)}{(2+\sigma)(2-\sigma^2)} < 0. \tag{40}$$

Therefore, the total production always decreases after privatization of firm 1, and hence, so does the consumer surplus.

Let us turn to the social welfare. Using (20) and (36), subtracting the social welfare under government management, SW^{G0} ($= SW^{O0}$), from that under privatization, SW^{P0}, we have,

$$SW^{P0} - SW^{G0} = -\frac{(1-c)^2\left[(1-\sigma)^2(3+2\sigma)+1\right]}{2(2+\sigma)^2(2-\sigma^2)^2} < 0. \tag{41}$$

That is, the social welfare is higher under government management than under privatization, and privatization is not desirable. The model here is an ordinary mixed duopoly and the results above are well known.

3.2 When investments are possible

Now we take into account the effects of cost-reducing investments, and compare the amounts produced and the social welfares. Does the public firm in mixed duopoly (i.e., under government management and outsourcing) produce more than the private firm as in Section 3.1? From (17) the difference in production by the two firms are as follows:

$$x_1^G - x_2^G = \frac{-(1-c)\left[2-\delta(2-\sigma^2)\right]}{-2+\delta(2-\sigma^2)^2}, \tag{42}$$

where the denominator is positive from the condition (16). Therefore, in order for $x_1^G > x_2^G$ to hold, the following condition is necessary:

$$\delta > \frac{2}{2-\sigma^2}. \tag{43}$$

Which condition is stricter, (16) or (43)? Let $\bar{\delta} \equiv 2/(2-\sigma^2)^2$ and $\hat{\delta} \equiv 2/(2-\sigma^2)$. Then we have,

$$\bar{\delta} - \hat{\delta} = -\frac{2(1-\sigma^2)}{(2-\sigma^2)^2} < 0. \tag{44}$$

That is, $\bar{\delta} < \hat{\delta}$ and hence the condition (43) is stricter. From the discussions above we obtain the following proposition:

Proposition 1. *If δ satisfies $\bar{\delta} < \delta < \hat{\delta}$ we have $x_1^G < x_2^G$. If $\hat{\delta} \le \delta$ we have $x_1^G \ge x_2^G$.*

Proposition 1 means that if δ satisfies (16) but is small enough (i.e., the investment cost is low enough) the private firm produces more than the public firm. If δ is large enough and satisfies (43) the public firm produces more than the private firm. Figure 1 shows the case where $c = 0.3$ and $\sigma = 0.3$. In this case we have $\bar{\delta} \approx 0.55$ and $\hat{\delta} \approx 1.05$. The curve of x_1^G consists of three parts. First, we do not consider the part where $\delta \le \bar{\delta}$ as it does not satisfy (16). Second, it follows from (17) that $x_1^G < 0$ where $\bar{\delta} < \delta < 2/(2-\sigma)(2-\sigma^2)$. Since it is not feasible we have $x_1^G = 0$ as a corner solution, and only firm 2 produces goods.[4] Third, (17) applies where $\delta > 2/(2-\sigma)(2-\sigma^2)$. From (17) we have,

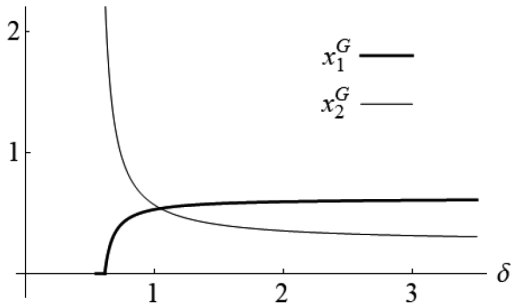

Figure 1: Graph of x_1^G and x_2^G

$$\frac{\partial x_1^G}{\partial \delta} = \frac{2(1-c)(1-\sigma)\sigma\left(2-\sigma^2\right)}{\left[-2+\delta\left(2-\sigma^2\right)^2\right]^2} > 0,$$

and,

$$\lim_{\delta\to\infty} x_1^G = \frac{(1-c)(2-\sigma)}{\left(2-\sigma^2\right)}, \tag{45}$$

that is, x_1^G is increasing in δ and converges to $(1-c)(2-\sigma)/(2-\sigma^2)$.

Next, using (17) and (33), we would like to see if the privatized firm produces more than it did before privatization. Subtracting x_1^G from x_1^P we have,

$$x_1^P - x_1^G = \frac{2(1-c)\left[4-\delta(2-\sigma)\left(6+3\sigma-\sigma^2\right)+\delta^2(2-\sigma)(2+\sigma)\left(2-\sigma^2\right)\right]}{\left[4-\delta(2-\sigma)(2+\sigma)^2\right]\left[-2+\delta\left(2-\sigma^2\right)^2\right]}. \tag{46}$$

Unfortunately, one cannot tell the sign of (46) analytically.[5] From (33), however, we have,

$$\frac{\partial x_1^P}{\partial \delta} = -\frac{4(1-c)(2-\sigma)(2+\sigma)}{\left[-4+\delta(2-\sigma)(2+\sigma)^2\right]^2} < 0,$$

and,

$$\lim_{\delta\to\infty} x_1^P = \frac{1-c}{2+\sigma}, \tag{47}$$

that is, x_1^P is decreasing in δ and converges to $(1-c)/(2+\sigma)$. From (45) and (47) we have,

$$\lim_{\delta\to\infty} x_1^G - \lim_{\delta\to\infty} x_1^P = \frac{2(1-c)}{(2+\sigma)\left(2-\sigma^2\right)} > 0. \tag{48}$$

Therefore, the curves of x_1^G and x_1^P must intersect. The discussions above are summarized as follows:

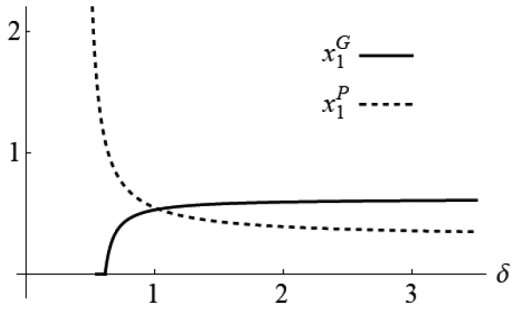

Figure 2: Graph of x_1^G and x_1^P

Proposition 2. *If δ is small enough we have $x_1^G < x_1^P$. Otherwise we have $x_1^G \geq x_1^P$.*

Figure 2 shows the case where $c = 0.3$ and $\sigma = 0.3$, as in Figure 1. One can see, as Proposition 2 says, that the privatized firm produces more than it did before privatization if the investment cost is low enough.

Now let us compare the social welfare when firm 1 is under government management, SW^G, with that under outsourcing, SW^O, and privatization, SW^P. From (18) and (24) we have,

$$SW^G - SW^O = -\frac{(1-c)^2\left[2-\delta\left(2-\sigma\right)\left(2-\sigma^2\right)\right]^2}{2\delta\left[2-\delta\left(2-\sigma^2\right)^2\right]^2} < 0.$$

That is, if outsourcing is available it is always more desirable than government management. The signs of $SW^G - SW^P$ and $SW^O - SW^P$ are, however, ambiguous. Therefore, we again set parameters c and σ, and plot the welfares as functions of δ. Figure 3 shows the case where $c = 0.3$ and $\sigma = 0.1$, i.e., the substitutability of the two goods is low. One can see that SW^P is the highest where δ is small enough. That is, the society is better off under privatization if the investment cost is low enough. As δ increases, however, SW^O exceeds SW^P. Therefore, outsourcing is the most desirable. SW^P is the lowest where δ is large enough because the cost reduction from the investment becomes very small and the advantage from maximizing the government's objective function becomes dominant. In other words, there is a trade-off between cost reduction of ancillary services and public firm's maximization of (incomplete) social welfare.

Looking back at the history of privatization, approaches such as outsourcing were introduced first, followed by privatization (or concession-based PPP). This can be attributed to a decline in investment costs due to technological innovations, in addition to the emergence of financial techniques such as project financing.

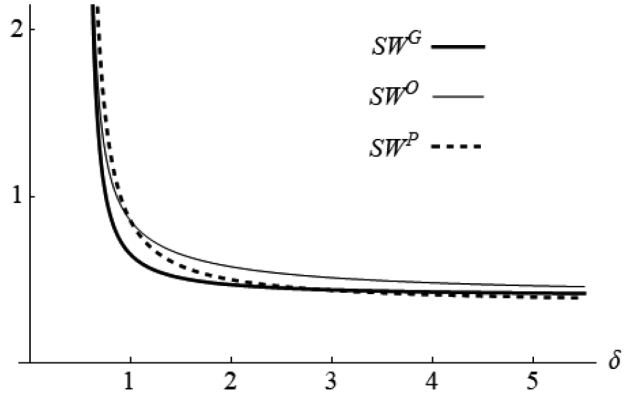

Figure 3: Welfares ($\sigma = 0.1$)

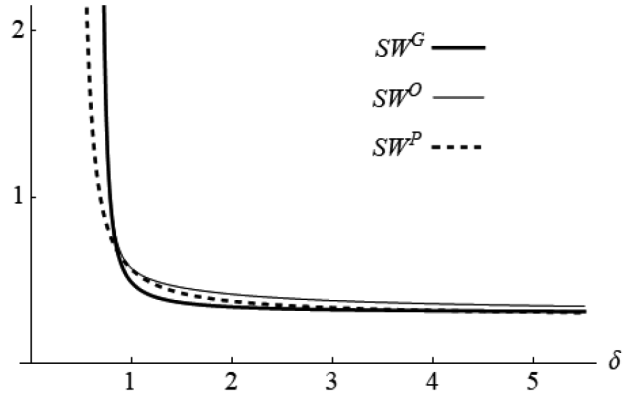

Figure 4: Welfares ($\sigma = 0.5$)

Things change when the substitutability of the two goods increases. Figure 4 shows the case where $c = 0.3$ and $\sigma = 0.5$. In this case SW^P is always lower than SW^O. Therefore, outsourcing is always the most desirable. For each firm, as σ increases from 0 to 1, the market shifts from a near-monopoly to a duopoly of homogeneous goods. Thus, if σ is large, privatization of firm 1 has a larger impact on firm 2. If δ is small enough firm 2 reduces production significantly, and the consumer surplus decreases.[6)] Therefore, the social welfare is lower under privatization than under outsourcing.

From the discussions above, we obtain the following proposition.

Proposition 3. *The public firm should be privatized if both the substitutability of the two goods and the investment cost are low enough.*

If σ is small enough the two goods are largely unrelated to each other and each firm is a near monopoly. Therefore, the effects as in contracting and bundling/unbundling literature are dominant, and privatization can be more desirable. Otherwise, privatization is not desirable as in

Table 1: Values of variables where $\delta = 0.9$

	x_1	x_2	v_1	v_2	p_1	p_2	CS	Π_1	Π_2	SW
gov't management	0.63	0.72	0.00	0.81	0.30	0.22	0.50	0.00	0.23	0.73
outsourcing	0.63	0.72	0.70	0.81	0.30	0.22	0.50	0.22	0.23	0.95
privatization	0.71	0.71	0.79	0.79	0.22	0.22	0.55	0.22	0.22	1.00

Table 2: Values of variables where $\delta = 2$

	x_1	x_2	v_1	v_2	p_1	p_2	CS	Π_1	Π_2	SW
gov't management	0.66	0.42	0.00	0.21	0.30	0.51	0.33	0.00	0.13	0.47
outsourcing	0.66	0.42	0.33	0.21	0.30	0.51	0.33	0.11	0.13	0.58
privatization	0.44	0.44	0.22	0.22	0.52	0.52	0.21	0.14	0.14	0.50

the standard mixed duopoly model.

3.3 Numerical examples

Now let us see the numerical examples of the three regimes, government management, outsourcing, and privatization. Table 1 shows the case where $c = 0.3$, $\sigma = 0.1$ and $\delta = 0.9$. In this case the parameter of the investment cost δ does not satisfy the condition (43) and the private firm (firm 2) produces more than the public firm (firm 1) in mixed duopoly. On the other hand, firm 1's production increases after privatization because δ is small enough. The social welfare is the highest under privatization.

Table 2 shows the case where $c = 0.3$, $\sigma = 0.1$ and $\delta = 2$. Now the condition (43) is satisfied, and hence the public firm produces more than the private firm in mixed duopoly (and so v_1 is larger than v_2 under outsourcing). On the other hand, firm 1's production decreases after privatization because δ is large enough. The investment levels are generally lower than in Table 1. The social welfare is the highest under outsourcing.

In Table 1, the total production increases after privatization of firm 1, and hence, so does the consumer surplus. In addition, the sum of v_1 and v_2 is larger under privatization, which helps enhance the social welfare. In table 2, however, the total production decreases after privatization. The sum of v_1 and v_2 also decreases, which lowers the social welfare under privatization.

4. Conclusion

While privatization takes various forms in these decades, the theory of mixed oligopoly does not fully reflect it. In addition, in standard models of mixed duopoly, the public firm produces more than the private firm in order to maximize the social welfare. It decreases production after privatization in order to maximize profit, and the social welfare is reduced.

They are, however, not necessarily the case in reality.

In the present paper we introduced *ex ante* investments for cost reduction of the ancillary services and the choice of bundling or unbundling of tasks into the model of differentiated mixed duopoly. We assumed that the public firm does not take into account the profits of ancillary services of the two facilities because of its inability to make good use of profit-making ancillary services. If one assumed that the public firm is all-knowing, the results would be similar to those of the previous studies. We showed that if the investment cost is low enough, the private firm produces more than the public firm in mixed duopoly, and the privatized firm produces more than it did before privatization. If, in addition, the substitutability of goods is also low enough, the social welfare is higher under privatization than under government management and outsourcing. The results above can explain public and private firms in the real world better than previous studies. The present study also clarified the mechanism by which privatization enhances social welfare by encouraging investment in ancillary services. These results can provide a theoretical basis for introducing PPPs to public services in mixed oligopolistic markets. The government may need to implement policies that support technological innovation to reduce private investment costs through tax systems or other measures.

In the present paper we considered a cost-reducing investment. We could examine the effects of a quality-enhancing investment. It is left for future research.

Appendix

In section 3.2 one may wonder what would it be if σ equals unity, or the two goods are perfect substitutes. Substituting $\sigma = 1$ into (18) we have $SW^G = (1 - c)^2/2$, which is a constant. SW^O and SW^P simplify to $(1 - c)^2 (1 + \delta)/2\delta$ and $[4\delta(1 - c)^2]/(-4 + 9\delta)$, respectively. Substituting $\sigma = 1$ into (15), (17), (22), and (23) we have $v_2 = x_2 = 0$ under government management and outsourcing, and only the public firm produces goods.

Figure 5 shows the case where $c = 0.3$ and $\sigma = 1$. Then it seems from Figure 5 that privatization is again the most desirable if δ is small enough. Unfortunately, from (16) we have $\delta > 2$ under government management and outsourcing, and hence the curves of SW^G and SW^O where $\delta \leq 2$ are invalid. Although from (32) we have $\delta > 4/9$ under privatization, one cannot compare the welfare with other regimes where $\delta \leq 2$. Therefore, outsourcing is still the most desirable where (16) is satisfied.

Oshima (2024) used a mixed duopoly model with *ex ante* investments where homogeneous goods and increasing marginal cost for production are assumed. The result is that privatization is the most desirable when the investment cost is low enough. Which result is more plausible when the goods are homogeneous depends on whether the marginal cost for production of the goods in question is closer to constant or to increasing.

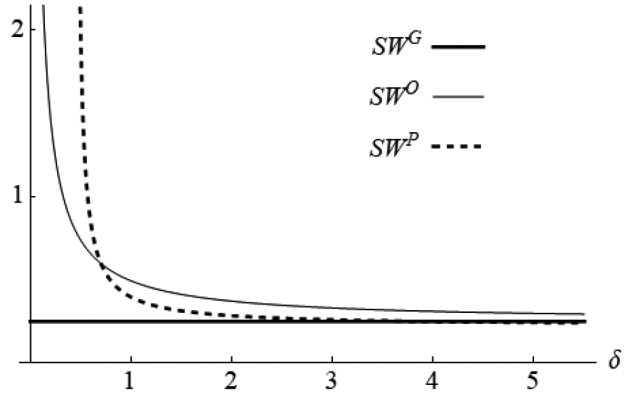

Figure 5: Welfares ($\sigma = 1$)

Acknowledgments

An earlier version of this paper was presented at Japanese Economic Association 2024 Spring Meeting held at Tokyo Keizai University. I would like to thank Tsuyoshi Shinozaki and other participants for helpful comments.

Notes

1) Since we consider two competing firms, we do not assume complementarity between the two goods just as in many earlier studies. We here focus on the consumption of core business goods.
2) The overall profit of facility 1, Π_1, or the sum of the profits of firm 1 and the ancillary services minus investment cost, would be given by $\Pi_1 \equiv \pi_1 + v_1 x_1 - \delta v_1^2/2$.
3) This condition arises because both π_2 and $-\delta v_2^2/2$ in Π_2 are quadratic functions of v_2. One may interpret (16) as the lower limit of cost reduction.
4) While the equilibrium does change at the corner solution, the effect is small and is not discussed in detail in the present paper. Graphically speaking, the graph of x_2^G in Figure 1 merely shifts slightly in the region above the graph's range. Hence, the Propositions are not affected.
5) The sign of $(x_1^P + x_2^P) - (x_1^G + x_2^G)$ is also ambiguous.
6) If both σ and δ are large firm 2 increases production when firm 1 is privatized, but firm 1 further reduces production. Thus the consumer surplus still decreases.

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（要旨）

混合寡占モデルは民営化の効果を研究する手法の一つである。混合複占の標準的なモデルでは，公企業は社会厚生を最大化するため民間企業よりも多く生産する。また，公企業は民営化されると生産を減らし，社会厚生は減少する。だが現実には必ずしもそうならない。今日，民営化は官民協働（PPP）など様々な形態を取るが，混合寡占の理論はそれを十分に反映していない。本論文では付随的業務の費用削減のための事前投資と業務のバンドリング／アンバンドリングの選択を差別化混合複占のモデルに導入する。公営，アウトソーシング，民営化の3つのレジームを考え，以下を示す。費用削減のための投資費用が十分に低ければ民間企業は公企業よりも多く生産し，公企業を民営化すると民営化前よりも生産を増やす。加えて2財の代替性も十分に低ければ民営化の下で社会厚生は最大になる。そうでなければアウトソーシングが最も望ましくなる。