Differentiated Mixed Oligopoly, Free Entry, and Decentralization in a Two-City Model

2都市モデルにおける差別化混合寡占、自由参入、および地方分権

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(Abstract)
This paper investigates the effects of centralization and decentralization in a model of differentiated mixed oligopoly with two cities. We find that, under centralization where national government owns the public firm, the optimal level of privatization is mountain-shaped in substitutability of varieties of the differentiated good when the number of private firms is fixed (short-run case), while it is monotonically decreasing and approaches zero when private firms can enter and exit the market freely (long-run case), which are as in earlier studies. On the other hand, under decentralization where city government owns the public firm, the privatization level reaches zero where the substitutability of varieties is less than unity. In addition, in the long-run case it shifts downward as marginal and/or fixed costs of production increases. This is because the city government does not take into account the profits of private firms in the other city, which leads to an inefficient level of privatization.
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1. Introduction

Privatization has been an important issue for policy makers as well as economists for decades, and hence many theoretical studies have been conducted. Mixed oligopoly is one of useful approaches to investigate privatization. It is a market where a small number of firms, public and private, compete with each other. While private firms maximize profits, public firms act in order to maximize social welfare if they are fully owned by the government. Even after a world wide trend of privatization many public firms still exist and are competing with private firms in various oligopolistic markets, and there are discussions about how the government should handle those firms. This is why we should pay attention to the studies of mixed oligopoly.

In their seminal paper De Fraja and Delbono (1989) showed that, in the mixed oligopoly with a fixed number of firms, the social welfare can be higher when the public firm maximizes its profit than when it aims to maximize welfare. This paradoxical result rests on the assumption of increasing marginal cost, and suggests that the public firm should be privatized in some cases. They considered the choice between two extremes, full nationalization and full privatization. There are firms, however, that are partially privatized and jointly owned by government and private party. Such firms have two targets, maximization of social welfare and maximization of their own profits, and are assumed to maximize the weighted average of these targets. Matsumura (1998) investigated partial privatization in mixed duopoly where a partially privatized public firm and a private firm compete in a market. He showed that partial privatization is more desirable than full nationalization or full privatization under plausible conditions. Matsumura and Kanda (2005) investigated the case where firms can enter the market freely. Private firms continue to enter the market until their profits equal zero. As a result, they showed that the public firm should produce goods so that the price equals its marginal cost, and in order to do so the government should fully own the public firm.

While the goods produced are homogeneous in the studies above, a number of studies introduced product differentiation and investigated its effects. Fujiwara (2007) showed that partial privatization is optimal except in extreme cases such that the products are homogeneous. The optimal privatization level in the short run is mountain-shaped in the degree of substitutability of products, while it is monotonically decreasing when private firms can enter the market freely.\(^1\)

As another direction to extend the model, researchers introduced multiple countries or regions. Bárcena-Ruiz and Garzón (2005) investigated a mixed oligopoly with two countries. There is a strategic interaction between two governments when they decide whether to privatize their public firms. They showed that when the marginal cost of public firms is high enough both governments privatize. On the other hand, only one government privatizes if the marginal cost of public firms takes an intermediate value. Dadpay and Heywood (2006) found that the governments of two countries face a prisoners’ dilemma when they decide...
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whether to privatize their public firms. That is, privatizing only one country’s public firm decreases that country’s welfare and increases that of the other country. Han and Ogawa (2008) allowed for partial privatization and investigated the effect of market integration. They showed that the governments are less eager to privatize in the international mixed oligopoly market than in a single-country framework.

Little attention has been paid, however, to the issues of centralization and decentralization in the literature of mixed oligopoly, while central and local governments may make different decisions in providing public goods and services. Examples of differentiated local goods and services include museums, tourist facilities, and local specialties. Among various examples, airports may be one of the most important goods and services. There are private, privatized, state-owned, and local-government-owned airports competing with neighboring ones. Oshima (2017), based on Fujiwara’s (2007) short-run case, considered a differentiated mixed duopoly in a two-city model where a public firm operate in one city and a private firm in the other, and compared decentralized and centralized solutions. The author found that the privatization level is higher under centralization if the substitutability of the differentiated good is not too low. In particular, unlike Fujiwara (2007), it is zero under decentralization if the substitutability is higher than a certain threshold.

The present paper extends Oshima’s (2017) model so that there are more than one private firms, and the number of firms are either fixed (short-run case) or endogenous (long-run case). It aims to investigate what effects centralization/decentralization has on the levels of privatization which the governments choose when there are more than one private firms. For this purpose we employ a quadratic utility function and a Cournot-Nash game as in earlier studies. Then we find that under centralization the results are generally in line with Fujiwara (2007) despite there are two cities. Under decentralization, in the short run the results are similar to those of Oshima (2017), while in the long run some interesting results are obtained. For example, the privatization level depends on a few parameters. It decreases as the fixed and/or marginal cost of production increases.

The rest of the paper is organized as follows. In Section 2 we set up a model. In Sections 3 and 4 we investigate the privatization levels which the governments choose under centralization and decentralization, respectively. Section 5 concludes.

2. The model

Suppose there are \(n+1\) firms in a country that consists of two cities, 0 and 1. Oshima (2017) assumed a mixed duopoly where a public firm is located in city 0 and a private firm in city 1. If more private firms enter the market we can consider the following three cases: (i) they enter into city 0, (ii) they enter into city 1, (iii) they enter into both cities. Case (i) is not much different from a single-city model. Case (iii) can be considered intermediate between cases (i) and (ii). Therefore in the present paper we assume case (ii) where private firms enter and are located in city 1.
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A public firm (firm 0) is located in city 0, and is owned by the government of city 0 (decentralization) or by the national government (centralization). Private firms (firms 1, 2, ..., n) are located in city 1. The firms produce a differentiated good. The public firm produces variety 0 of the good and the private firms produce varieties 1, 2, ..., n. In addition, a homogenous good which is a numeraire is produced in both cities.

The utility function of the representative consumer in city \(i\) is expressed as follows:

\[
 u^i = a \sum_{j=0}^{x^i_j} - \frac{1 - b}{2} \sum_{j=0}^{x^i_j} (x^i_j)^2 - \frac{b}{2} (\sum_{j=0}^{x^i_j})^2 + z, \quad i = 0, 1, \tag{1}
\]

where \(x^i_j\) is the amount of variety \(j\) of the differentiated good consumed in city \(i\), and \(z\) is the amount of homogeneous good. We assume \(a > 0\) and \(b \in [0,1)\). Parameter \(b\) shows the substitutability of the differentiated good. \(^3\)

Let \(p_j, j = 0, 1, ..., n\) denote the price of variety \(j\). Given the budget constraint \(I = \sum p_j x^i_j + z\), solving the maximization problem of consumers in two cities we have,

\[
 p_k = a - (1 - b)x^i_k - b \sum_{j=0}^{x^i_j}, \quad i = 0, 1. \tag{2}
\]

Summing up (2) of the two cities and rearranging yields the inverse demand function for variety \(k\):

\[
 p_k = a - \frac{1 - b}{2} (x^0_k + x^1_k) - \frac{b}{2} (\sum x^0_j + \sum x^1_j). \tag{3}
\]

Using (1) and (3) we obtain the consumer surpluses in two cities as below (in what follows we take summation over \(n\) varieties produced by private firms) :

\[
 CS^0 = u^0 - p_0 x^0_0 - \sum p_j x^0_j - z = \frac{1 - b}{2} (x^0_0 x^1_0 + \sum x^0_j x^1_j) - \frac{b}{2} (x^0_0 + \sum x^0_j)(x^1_0 + \sum x^1_j) = CS^1. \tag{4}
\]

Now we consider the producers. Suppose that the firms face the same technology and the cost function \(C_j = c(x^0_j + x^1_j) + f, j = 0, 1, ..., n\) where \(c\) is the marginal cost, \(c < a\), and \(f\) is the fixed cost, which are usual assumptions in the literature. \(^4\) Then the profit of firm 0 is as follows:

\[
 \pi_0 = p_0(x^0_0 + x^1_0) - c(x^0_0 + x^1_0) - f = (a - c - \frac{x^0_0 + x^1_0}{2} - \frac{b}{2} (\sum x^0_j + \sum x^1_j))(x^0_0 + x^1_0) - f. \tag{5}
\]
Similarly, for the profit of firm \( k \) we have,

\[
\pi_k = \{a - c - \frac{1 - b}{2}(x_k^0 + x_k^1) - \frac{b}{2}(x_0^0 + \sum x_j^0 + x_0^1 + \sum x_j^1)](x_k^0 + x_k^1) - f. \tag{6}
\]

In what follows we analyze two solutions, centralized and decentralized ones, in each of which we investigate short-run (fixed number of firms) and long-run (free entry and exit) cases where the firms play Cournot-Nash games. The game consists of two stages. In the first stage the government chooses the privatization level of its public firm. In the second stage the firms determine the quantities supplied. In the long-run cases private firms enter the market until their profits equal zero before production in the second stage. Then we solve the game by backward induction.

3. Centralization

Suppose that the national government owns the public firm and maximizes the welfare of the country defined as below:

\[
W = CS^0 + CS^1 + \pi_0 + \sum \pi_j. \tag{7}
\]

The public firm maximizes the weighted average of its profit and the welfare of the country:

\[
\max \theta \pi_0 + (1 - \theta)W = \pi_0 + (1 - \theta)(CS^0 + CS^1 + \sum \pi_j),
\]

where \( \theta \in [0,1] \) is the level of privatization. \( \theta = 0 \) means that the public firm is fully owned by the government while it is fully privatized if \( \theta = 1 \). Assume symmetry among private firms such that they produce the same amount \((x_i = x_j)\). Then, using (4) – (6), we obtain the first-order condition as follows,

\[
a - c - (x_0^0 + \theta x_0^1) - \frac{nb}{2} [(2 - \theta)x_1^0 + \theta x_1^1] = 0. \tag{8}
\]

On the other hand, private firms maximize \( \pi_k \). Using (6), the first-order condition is,

\[
a - c - \frac{b}{2}(x_0^0 + x_1^0) - \frac{2 + b(n - 1)}{2}(x^1_0 + x_1^1) = 0. \tag{9}
\]

From (2), (8), and (9) we have the equilibrium consumption of varieties in two cities as follows:
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\[ x_0^C = x_1^C = \frac{(a - c)(2 - b)}{[2 + b(n - 1)](1 + \theta) - nb^2} \]

where the superscript \( C \) denotes centralized equilibrium.

3.1 Short-run case

In this subsection we suppose that the number of private firms, \( n \), is fixed. Substituting (10) into (4) – (6), from (7) we obtain the welfare of the country as a function of \( \theta \), \( W(\theta) \). The first-order condition is as follows,

\[ W'(\theta^{CS}) = \frac{2(2 - b)(a - c)^2[4\theta^{CS} - b(n + (4 - n)\theta^{CS}) + b^2((n + (1 - n)\theta^{CS})]}{[-(1 + \theta^{CS})(2 + b(n - 1)) + nb^2]^3} = 0, \tag{11} \]

where \( \theta^{CS} \) is the optimal level of privatization under centralization in the short run. Solving (11) for \( \theta^{CS} \) yields,

\[ \theta^{CS} = n b(1 - b) \frac{1}{nb(1 - b) + (b - 2)^2}. \tag{12} \]

Differentiating (12) with regard to \( n \) we have,

\[ \frac{\partial \theta^{CS}}{\partial n} = \frac{b(1 - b)(2 - b)^2}{(nb(1 - b) + (b - 2)^2)^2} > 0, \]
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that is, the optimal privatization level is higher as the number of private firms increases. From (12) one can see that $\theta^{CS} = 1$ when $n \rightarrow \infty$.

Given the number of private firms, when is $\theta^{CS}$ the largest? The first-order condition of (12) with respect to $b$ is,

$$\frac{\partial \theta^{CS}}{\partial b} = \frac{n(2-3b)(2-b)}{\{nb(1-b) + (b-2)^2\}^2} = 0.$$ 

Therefore $\theta^{CS}$ is the largest when $b = 2/3$. $\theta^{CS}$ is depicted as in Figure 1. Substituting (12) into (10) we have,

$$x_0^{0CS} = x_0^{1CS} = \frac{(a-c)(4+b(n-4) - b^2(n-1))}{4 + b(4-b^2)(n-1) - b^2(n^2-5n+1)},$$

$$x_1^{0CS} = x_1^{1CS} = \frac{(a-c)(1-b)(2+b(n-1))}{4 + b(4-b^2)(n-1) - b^2(n^2-5n+1)}.$$ 

3.2 Long-run case

Suppose now that private firms can enter and exit the market freely. That is, the number of private firms, $n$, is endogenously determined and the profits of private firms equal zero. Substituting (10) into (4) – (6) again, and solving $\pi_k = 0$ for $n$ yields,

$$n^{CL} = \frac{\sqrt{2f(a-c)(1+\theta-b) - (2-b)(1+\theta)}}{b(1+\theta-b)},$$

where the superscript CL denotes the equilibrium under centralization in the long run. Substituting (14) into (10) we have,

$$x_0^{0CL} = x_0^{1CL} = \frac{f}{2} \frac{2-b}{1+\theta-b}, \quad x_1^{0CL} = x_1^{1CL} = \frac{f}{\sqrt{2}}.$$ 

Substituting (15) into (4) and (5) and using (7) we obtain $W(\theta)$ in the long-run case. The first-order condition is as follows:

$$W'(\theta^{CL}) = \frac{f(2-b)(1 - 3\theta^{CL} - b(2 - \theta^{CL}) + b^2)}{2(1 + \theta^{CL} - b)^3} = 0,$$

where $\theta^{CL}$ is the optimal privatization level in the long-run case. Solving (16) for $\theta^{CL}$ yields,
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\[ \theta^{CL} = \frac{(1 - b)^2}{3 - b}, \]

which is depicted as in Figure 2. As in Fujiwara (2007) it is independent of fixed and marginal costs of production, \( f \) and \( c \), but just depends on \( b \). Differentiating (17) with regard to \( b \) yields,

\[ \frac{\partial \theta^{CL}}{\partial b} = \frac{(5 - b)(1 - b)}{(3 - b)^2} < 0. \]

That is, \( \theta^{CL} \) is monotonically decreasing in \( b \) in the domain \( b \in [0, 1) \). It is the highest (\( \theta^{CL} = 1/3 \)) when \( b = 0 \), while it is zero (full nationalization) if \( b = 1 \). Substituting (17) back into (15) we have,

\[ x_0^{0CL} = x_0^{1CL} = \frac{f}{\sqrt{2}} \frac{3 - b}{2(1 - b)}. \]

Raising \( \theta \) induces entry by private firms. One can confirm this by differentiating (14) with regard to \( \theta \) to obtain,

\[ \frac{\partial n^{CL}}{\partial \theta} = \frac{2 - b}{(1 + \theta - b)^2} > 0. \]

This has two opposite effects. First, it provides consumers with more varieties of the differentiated good. On the other hand, it can cause inefficiently excessive entry. As discussed in Fujiwara (2007), when the varieties of the good are so differentiated (\( b \) is close to zero) the former effect is dominant, and the government should raise \( \theta \) to induce entry by private firms. As \( b \) increases the more dominant is the latter effect, and the government should
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lower $\theta$ to reduce wasteful entry by private firms. $\theta^{ct}$ is the privatization level where the two effects are balanced.

4. Decentralization

The results in the previous section are in line with Fujiwara (2007) despite there are two cities in the present model. Now let us see the case where the government decision is decentralized. That is, the government of city 0 owns the public firm and maximizes the welfare of the city which is defined as follows:

$$W^0 \equiv CS^0 + \pi_0.$$ \hfill (19)

The public firm maximizes the weighted average of its profit and the welfare of city 0:

$$\max \theta \pi_0 + (1 - \theta)W^0 = \pi_0 + (1 - \theta)CS^0.$$  

Using (4) and (5) we obtain the first-order condition as below.

$$a - c - x_0^0 - \frac{1 + \theta}{2}x_0^1 - \frac{nb}{2}(x_0^0 + \theta x_1^1) = 0. \hfill (20)$$

We already have the first-order condition for private firms as (9). From (2), (9), and (20) we have the equilibrium consumption of two varieties in two cities as follows:

$$x_{0D}^0 = x_{0D}^1 = \frac{2(a - c)(2 - b)}{(2 + (n - 1)b)(3 + \theta) - 2nb^2}, \hfill (21)$$

$$x_{1D}^0 = x_{1D}^1 = \frac{(a - c)(3 - 2b + \theta)}{(2 + (n - 1)b)(3 + \theta) - 2nb^2},$$

where the superscript $D$ denotes decentralized equilibrium.

4.1 Short-run case

Given the number of private firms, substituting (21) into (4) and (5), from (19) we obtain the welfare of city 0 as a function of $\theta$, $W^0(\theta)$. Solving the first-order condition $W^0(\theta^{DS}) = 0$ for $\theta^{DS}$ we have:

$$\theta^{DS} = \frac{nb\{3 + 3(n - 3)b - (3n - 4)b^2\}}{8 + (11n - 8)b - (3n - 2)nb^3 + (3n^2 - 9n + 2)b^2}, \hfill (22)$$

where $\theta^{DS}$ is the level of privatization which the government of city 0 chooses in the short
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run, and is depicted as in Figure 3. It is negative when $b$ is large enough, which is, however, not feasible in reality. Therefore we set $θ^{DS}=0$ in such cases. While the sign of $\frac{∂θ^{DS}}{∂n}$ is ambiguous, one can see from Figure 3 that, as $n$ increases, the level of privatization generally shifts upward in the range $θ^{DS} \in [0,1]$. In addition, the domain where full nationalization is desirable ($θ^{DS}=0$) becomes smaller. Substituting (22) into (21) we have,

$$x_0^{0DS} = x_0^{1DS} = \frac{(a-c)(6 - 7n)b + (2 - 3n)b^2}{12 + 3(nb^2 - 4)(1 - n)b + (3 - 11n + 3n^2)b^2},$$

Unlike $θ^{CS}$, $θ^{DS}$ reaches zero where $b<1$ because, as discussed in Oshima (2017), the government of city 0 does not take into account the consumer surplus and firms' profits in city 1, and hence begins to lower $θ$ at smaller value of $b$ than under centralization.

4.2 Long-run case

Suppose again that $n$ is endogenously determined and the profits of private firms equal zero. Substituting (21) into (6) and solving $π_1 = 0$ for $n$ yields,

$$n^{DL} = \sqrt{\frac{2f(a-c)(3 + θ - 2b) - (2 - b)(3 + θ)}{(3 + θ)b - (1 + θ)b^2}},$$

where the superscript DL denotes the equilibrium under decentralization in the long run. Substituting (24) into (23) we have,

$$x_0^0 = x_0^1 = \frac{(a-c)(1 - θ) + \sqrt{f/2(2 - b)(1 + θ)}}{3 + θ - (1 + θ)b}, \quad x_1^0 = x_1^1 = \sqrt{\frac{f}{2}}.$$
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Then, substituting (25) into (4) and (5) and using (19) we obtain \( W^0(\theta) \) in the long-run case. Solving the first-order condition \( W^0(\theta_{DL}) = 0 \) for \( \theta_{DL} \) we have,

\[
\theta_{DL} = \frac{12(a-c)^2(1-b) - \sqrt{2f(a-c)(5-3b)(3+b)}}{(1-b)[12(a-c)^2 - \sqrt{2f(a-c)(11-3b) + (5-3b)f}]}. \tag{26}
\]

From (26) one can obtain some interesting results which are different from Fujiwara (2007). First, \( \theta_{DL} \) depends on a few parameters and one cannot see its characteristics analytically or draw its curve as it is. As one can see from (26), however, \( \theta_{DL} \) is unity regardless of other parameters if \( f=0 \). That is, if no fixed cost is required full privatization is always desirable.\(^9\) Suppose next that \( f \) is positive, \( a=1 \), and \( c=0.1 \). Figure 4 shows that \( \theta_{DL} \) is decreasing in \( b \) (downward-sloping) and reaches zero where \( b<1 \). In addition, it shifts downward as \( f \) increases, and eventually \( \theta_{DL} \) should equal zero regardless of \( b \). That is, full government ownership is always desirable.

Given other parameters, how does \( \theta_{DL} \) change when the marginal cost, \( c \), varies? Suppose \( a=1 \) and \( f=0.1 \), and see \( \theta_{DL} \) when \( c \) varies. Figure 5 shows that \( \theta_{DL} \) is less than unity and decreasing in \( b \) even if \( c=0 \). In addition, it shifts downward as \( c \) increases, and eventually \( \theta_{DL} \) should equal zero regardless of \( b \).

Why does the downward shift of \( \theta \) occur only under decentralization and not under centralization? One could interpret it as follows. Under decentralization the government of city 0 does not take into account the profits of private firms entering the market. It just takes care of the consumer surplus of its residents and the profit of its public firm. Therefore, the second effect of raising \( \theta \) mentioned in Section 3 is underestimated, and \( \theta_{DL} \) can be inefficiently high when the marginal and/or fixed cost is low. As the costs increase, the government lowers \( \theta_{DL} \) and increases the production of the public firm in order to prevent a drastic decrease in the consumer surplus because of a rise in prices. From (14) in Section 3 one can see that higher marginal and fixed costs reduce entry by private firms. That is, they
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serve as barriers to entry. Therefore the central government which takes all the firms into account does not need to lower the privatization level $\theta_{CL}$.

In reality, the central government (or the government to whose jurisdiction private firms enter) may not fully recognize the profits of private firms. Then the government may act to some extent like the city government in this subsection and lower the privatization level when the production costs increase.

5. Conclusion

In the present paper we investigated a differentiated mixed oligopoly with two cities, and compared centralized and decentralized solutions. Under centralization the results were generally in line with Fujiwara (2007), and in the short-run case under decentralization the results were similar to those of Oshima (2017). On the other hand, in the long-run case under decentralization some interesting results were obtained. The privatization level which the city government chooses depends on a few parameters. It is unity (full privatization), however, if the fixed cost of production equals zero. If the fixed cost is positive the privatization level is decreasing in substitutability of varieties, and it is zero (full government ownership) if the substitutability is large enough. The curve of the privatization level shifts downward as the fixed and/or marginal cost increases.

In the present paper we did not compare welfares under centralization and decentralization because the model was too complicated to compare the welfares analytically. In Oshima’s (2017) duopoly model, however, social welfare is always higher under centralization while the welfare of city 0 is higher under decentralization. This suggests that a (partially privatized) public firm should be owned by the national (or regional) government rather than the city government, but that the transition from decentralization to centralization would be politically difficult.

As next steps one can make some extensions to the model. For example, one may introduce more than two cities. Another example might be to consider multiple governments competing with each other. They are left for future research.

Appendix

In Section 4.1 the first-order condition for the government is as follows:

$$W^{0\prime}(\theta^{DS}) = \frac{A}{\{(1 + \theta^{DS})nb^2 - 2(3 + \theta^{DS}) - (n - 1)(3 + \theta^{DS})b\}^3} = 0,$$

where,
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\[ A \equiv -2(2-b)(a-c)^2(1+nb)[4 - 3(1-\theta^{DS})n - 2\theta^{DS}] \\
+ 3(n-3)(1-\theta^{DS})n - 2\theta^{DS} - 8\theta^{DS}(1-b) + (3-11\theta^{DS})nb]. \]

Solving this for \( \theta^{DS} \) yields (22).

In Section 4.2 the first-order condition for the government is as follows:

\[ W^{0'}(\theta^{DL}) = \frac{B}{2(3 + \theta^{DL} - (1 + \theta^{DL})b)} = 0, \]

where,

\[ B \equiv (2-b)[12(a^2 + c^2)(1-b)(1-\theta^{DL}) + f(9 - 5\theta^{DL} - 3(1+\theta^{DL})b^2) \\
+ 2(1 + 4\theta^{DL})b - \sqrt{2f(a-c)(15 - 11\theta^{DL} - 2(2-7\theta^{DL})b - 3(1+\theta^{DL})b^2)} \\
- 24ac(1-b)(1-\theta^{DL})]. \]

Solving this for \( \theta^{DL} \) yields (26).

Notes

1) See also Anderson et al. (1997), Saha (2009) and Choi (2012) for product differentiation.

2) In the present model we do not assume transportation cost to avoid complexities. See Oshima (2017) for a model with transportation cost which lowers the privatization level.

3) When \( b \rightarrow 1 \) only the aggregate consumption \( \Sigma x_i \) matters, i.e., the goods are perfect substitutes. See Vives (2001, sec. 6.1), for example. We here assume \( b<1 \) so that consumers purchase all varieties. While we use a simplified version of quadratic utility function with regard to parameters, there is no qualitative impact on the results.

4) We assume, as several earlier studies, that privatization does not improve firm’s efficiency.

5) Notice that \( \pi_k = 0, k=1,2, ..., n \).

6) The latter effect is closely related to the excess entry theorem. For the discussions on excess entry theorem see Mankiw and Whinston (1986), Matsumura (2000) and Matsumura and Kanda (2005).

7) See Appendix for the first-order condition.

8) In the present paper we omit the analyses of those cases to focus on privatization levels. See Oshima (2017) for such analyses.

9) In that case, from (24) the number of private firms entering the market would be infinite, though.
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References
Choi, K. (2012) "Cournot and Bertrand competition with asymmetric costs in a mixed duopoly revisited," MPRA Paper, No. 37004

（要旨）
本論文では消費財の差別化を考慮した２都市からなる混合寡占モデルにおいて、中央集権化と地方分権化の効果を分析する。中央政府が公企業を所有する中央集権下では、複数都市や地域のない1国モデルの先行研究と同様、民間企業の数が所与である短期においては民営化水準のグラフは財の代替性について山型となり、民間企業が参入・退出を自由にできる長期においては民営化水準は単調に減少しゼロへと近づく。ところが都市政府が公企業を所有する地方分権の下では、特殊なケースを除く短期・長期とも財の代替性が1（財は同質）よりも小さい値で民営化水準がゼロに落ちてしまう。さらに長期の場合、生産に要する固定費用や限界費用が増加すると民営化水準が下方にシフトする。これらは、都市政府が他の都市に参入する民間企業の利潤を考慮せず、価格上昇による消費者余剰の急減を防ぐために民営化水準を下げて公企業の生産を増加させるためと考えられる。