【Articles】

Mixed Duopoly, Privatization, and *ex ante* Investments

混合複占，民営化，及び事前投資

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Contents

(Abstract)

While privatization has taken various forms in these decades, including Public-Private Partnerships (PPPs), the theory of mixed oligopoly does not fully reflect it. In standard models of mixed duopoly, the public firm produces more than the private firm in order to maximize the social welfare. It decreases production after privatization in order to maximize its profit, and the social welfare is reduced. They are, however, not necessarily true in reality. In the present paper we introduce *ex ante* investments for cost reduction of ancillary services and the choice of bundling or unbundling of tasks into the model of mixed duopoly. We show that if the investment cost is low enough the private firm produces more than the public firm in mixed duopoly, and the privatized firm in pure duopoly produces more than it did before privatization. The social welfare is higher under privatization than under government's operation of the public firm if the investment cost is low enough. Lump-sum outsourcing of ancillary services is the most desirable if the investment cost is not low.

1. Introduction

Privatization has been an important issue for decades for many economists, as well as politicians and government officials. Mixed oligopoly is one of the approaches to investigate this issue. Mixed oligopoly is a market where a small number of firms, public and private, compete with each other. Many theoretical studies assume that private firms maximize profits while a public firm acts in order to maximize social welfare. If it is privatized it acts as a private firm and hence affects the social welfare. After the global trend of privatization many public firms still exist and are competing with private firms in various oligopolistic markets. There are discussions about how the government should handle those firms. This is one of the reasons why we need to study mixed oligopolies.

In their pioneering work De Fraja and Delbono (1989) showed that under certain conditions the welfare can be higher when the public firm maximizes its profit than when it aims to maximize welfare, and hence the public firm should be privatized. While they considered the choice between full nationalization and full privatization, Matsumura (1998) introduced partial privatization into the model of mixed duopoly where a partially privatized public firm, jointly owned by the government and a private party, compete with a private firm in the market. He showed that under plausible conditions neither full nationalization nor full privatization is desirable but partial privatization is optimal. Matsumura and Kanda (2005) considered free entry of firms into the market and showed that full nationalization is optimal. Fujiwara (2007) introduced product differentiation into the model and showed that partial privatization is optimal except in extreme cases. Han and Ogawa (2008) investigated the impact of the market integration of two countries and showed that the governments are less eager to privatize public firms in the international mixed oligopoly. Oshima (2018) considered a differentiated mixed duopoly in a two-city model where a public firm operates in one city and a private firm in the other, and showed that in most cases the privatization level is higher when the state owns the public firm than when the city owns the public firm.

There are, however, questions about the models of mixed oligopoly. In those models, a public firm reduces production when it is privatized in order to maximize profit. In reality, however, privatized firms in oligopolistic markets often increase production. In theory, privatization lowers the social welfare if the firms' marginal cost does not increase much or the number of firms is small. In reality, such privatizations have taken place and it is unlikely that they all lowered the social welfare.

Recently, the term privatization also refers to various forms of Public-Private Partnerships (PPPs) such as concession-based PPP, Private Finance Initiative (PFI), and others. In the case of concession-based PPP, unlike the partial privatization mentioned above, there is no transfer of ownership. Instead, the government sells the rights to operate the facility to a private firm or a consortium. Appropriate division of roles between the government and the private party is considered important. The concessionaire can make investments on behalf of

the government, which affects the result of the operation of the facility. Therefore, who makes investments matters. In the context of the theory of incomplete contracts, Hart (2003) defined PPP as a project where two tasks, facility construction and service provision, are bundled. That is, the government contracts with a private party to build and run the facility. On the other hand, under conventional provision, the government contracts with a builder to build the facility, and then later on with another private party to run it. The builder can make investments that affect the operation of the facility. Then he showed that PPP is desirable if the quality of the service can be well specified in the initial contract, whereas the quality of the building cannot be.

There is a group of studies which introduce R&D investments into the model of differentiated mixed duopoly. Their aim is to show under what conditions Cournot competition can be more efficient than Bertrand competition. Singh and Vives (1984) showed that the welfare of Bertrand competition is higher than that of Cournot competition in a differentiated duopoly where goods are substitutes. Qiu (1997) introduced *ex ante* R&D investment for cost reduction, which generates external economies for the other firm, into the model and showed that the result can be reversed. Symeonidis (2003) introduced the investment for quality enhancement of products into the model and showed that Cournot competition is more efficient under certain conditions.

In the present paper, we suppose two tasks, the core business and the ancillary services, for a project, and either the government, the contractors, or a PPP concessionaire can make investment for the ancillary services. We use the cost-reducing investment similar to Qiu's (1997). We are, however, not concerned with the superiority of Bertrand or Cournot competition. In addition, the investments in our model are not that of R&D and hence there is no external economy.

The rest of the paper is organized as follows. In section 2 we set up a model of mixed duopoly and derive the equilibria of the three regimes; government's operation of the public firm, lump-sum outsourcing of ancillary services, and privatization. In section 3 we compare the results of the three regimes. We also investigate the results where there is no technology for the cost-reducing investments. Section 4 concludes.

2. The model

Suppose that there are two identical facilities, facilities 1 and 2, which conduct two tasks; the core business which produces and sells the goods, and the ancillary services which bears no immediate relationship to producing the goods (e.g., restaurants and shops in an airport). The government owns facility 1 and a private firm owns facility 2. As mentioned in Section 1, the term privatization has various meanings. In the present paper, privatization refers to introducing the concession-based PPP, where a private consortium operates the whole facility (i.e., the core business and the ancillary services are bundled) and is delegated from the government the right to make investment which it thinks necessary before production. If the

facility is not privatized, the government outsources the ancillary services in the facility to private contractors (i.e., the core business and the ancillary services are unbundled), because of some reasons such that the government officials are not familiar with the business of the ancillary services. The government, however, does not delegate the right to make investment to those contractors because (the government may think that) such investments by the contractors may alter the facility and affect the core business (Section 2.1). If there is no such problem, the government can outsource the ancillary services to the alliance of contractors in a lump sum and delegate the right to make investment to the alliance (Section 2.2). If the two tasks are bundled, the operator of the facility can handle such problems and the government can delegate the right to make investment to a private consortium (Section 2.3).

Consider a three-stage game with two firms, firms 1 and 2, which operate the facilities mentioned above, respectively, and produce homogeneous goods. Firm 1 is a public firm owned by the government and firm 2 is a private firm, and hence the market is a mixed duopoly. In the first stage, the government decides whether to privatize the facility. In the second stage, each firm determines how much to make cost-reducing investment for the ancillary services. In the third stage, both firms produce and sell goods. We solve the game by backward induction.

Let p denote the price of the good and X the total amount produced. Suppose that the adverse demand function is described as below:

$$p = a - X, \quad a > 0 \tag{1}$$

Hence, the consumer surplus, CS, is described as $CS = X^2/2$. Let x_1 and x_2 denote the amounts produced by firms 1 and 2, and we have $X = x_1 + x_2$.

The two firms face the same cost function $cx_i^2/2$, $c > 0$, $i = 1,2$, for producing goods. Assume that firm i's profit of the ancillary services is described as sx_i, where the parameter s is the net marginal profit from the ancillary services. Suppose that the firms can make cost-reducing investments for the ancillary services at the second stage which reduce the marginal costs by v_i (v_i is given at the third stage). Therefore, the profit of the ancillary services is $(s + v_i)x_i$. For simplicity, we assume $s = 0$ below.

2.1 The government's operation of firm 1

Suppose that the government does not privatize the public firm. Then the public firm does not take into account the profit of the ancillary services. Thus the profits of the two firms net of investment costs are described as follows:

$$\pi_1 = px_1 - \frac{c}{2}x_1^2 = (a - x_1 - x_2)x_1 - \frac{c}{2}x_1^2, \tag{2}$$

$$\pi_2 = px_2 - \frac{c}{2}x_2^2 + v_2x_2 = (a + v_2 - x_1 - x_2)x_2 - \frac{c}{2}x_2^2. \tag{3}$$

The public firm does not take into account the ancillary services of firm 2, either. There-

fore, the objective function that the public firm maximizes, *GO*, is as follows:

$$GO = CS + \pi_1 + px_2 - \frac{c}{2}x_2^2 = a(x_1 + x_2) - \frac{1+c}{2}(x_1 + x_2)^2 + cx_1x_2. \tag{4}$$

Actually, it is not that the profits of the ancillary services disappear. The social welfare which is explained later includes those profits.

At the third stage, the public firm maximizes its objective function while the private firm maximizes its profit. From the first-order conditions we have,

$$\frac{\partial GO}{\partial x_1} = a - (1+c)x_1 - x_2 = 0, \tag{5}$$

$$\frac{\partial \pi_2}{\partial x_2} = a + v_2 - (2+c)x_2 - x_1 = 0. \tag{6}$$

Solving (5) and (6) for x_1 and x_2 we obtain,

$$x_1 = \frac{a(1+c) - v_2}{1 + 3c + c^2}, \quad x_2 = \frac{ac + (1+c)v_2}{1 + 3c + c^2}. \tag{7}$$

Substituting (7) into (2) and (3) we have,

$$\pi_1 = \frac{c[a(1+c) - v_2]^2}{2(1 + 3c + c^2)}, \tag{8}$$

$$\pi_2 = \frac{(2+c)[ac + (1+c)v_2]^2}{2(1 + 3c + c^2)}. \tag{9}$$

At the second stage, the two firms choose the levels of investment for the ancillary services. We assume that the investment cost for reducing the marginal cost by v_i is $\delta v_i^2/2$, $\delta > 0$. The investment cost increases with the parameter δ. Therefore, the social welfare, *SW*, which is the sum of the consumer surplus and firms' profits minus investment costs, is described as below:

$$SW = CS + \pi_1 + v_1x_1 + \pi_2 - \frac{\delta}{2}(v_1^2 + v_2^2). \tag{10}$$

On the other hand, the overall objective function for the public firm, *OGO*, is given by $OGO = GO - \delta v_1^2/2$. The overall profit of the private firm, Π_2, is given by $\Pi_2 = \pi_2 - \delta v_2^2/2$.[1)] From the first-order conditions we have,

$$\frac{\partial OGO}{\partial v_1} = -\delta v_1 = 0, \tag{11}$$

$$\frac{\partial \Pi_2}{\partial v_2} = \frac{ac(2+3c+c^2)+v_2[(1+c)^2(2+c)-\delta(1+3c+c^2)^2]}{(1+3c+c^2)^2} = 0. \tag{12}$$

Hence, solving (11) and (12), the levels of investment in this mixed duopoly, v_1^M and v_2^M, are as follows:

$$v_1^M = 0, \quad v_2^M = \frac{ac(1+c)(2+c)}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}. \tag{13}$$

In order for v_2^M to be positive the denominator needs to be positive, and hence the following condition is necessary:

$$\delta > \frac{(1+c)^2(2+c)}{(1+3c+c^2)^2}, \tag{14}$$

which we assume in what follows. If the condition (14) is not satisfied, firm 2 chooses $v_2^M = 0$ as a corner solution. We will see the case where $v_1 = v_2 = 0$ later.

Substituting (13) into (7) we have the amounts produced, x_1^M and x_2^M, as follows:

$$x_1^M = \frac{a(1+c)[-2-c+\delta(1+3c+c^2)]}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}, \tag{15}$$

$$x_2^M = \frac{ac\delta(1+3c+c^2)}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}. \tag{16}$$

Substituting (8), (9), (13), (15) and (16) into (10) we have the social welfare, SW^M as below:

$$SW^M = \frac{\Lambda}{2[(1+c)^2(2+c)-\delta(1+3c+c^2)^2]^2}, \tag{17}$$

where,

$$\begin{aligned}\Lambda \equiv a^2[&(1+c)^3(2+c)^2\\ &-(1+c)(2+c)(2+12c+24c^2+15c^3+3c^4)\delta\\ &+(1+3c+c^2)^2(1+5c+8c^2+2c^3)\delta^2].\end{aligned}$$

Suppose now that there is no such technology that can reduce the marginal cost of the ancillary services. That is, the market is an ordinary mixed duopoly and the firms do not make cost-reducing investments. Substituting $v_1 = 0$ and $v_2 = 0$ into (7) yields the amounts produced, x_1^{M0} and x_2^{M0}, as follows:

$$x_1^{M0} = \frac{a(1+c)}{1+3c+c^2}, \quad x_2^{M0} = \frac{ac}{1+3c+c^2}. \tag{18}$$

Then we have the social welfare, SW^{M0} as below:

$$SW^{M0} = \frac{a^2(1+5c+8c^2+2c^3)}{2(1+3c+c^2)^2}. \tag{19}$$

2.2 Lump-sum outsourcing of the ancillary services

Next let us assume that the government can outsource the ancillary services in a lump sum and delegate the right to make investment to the alliance of the contractors. In this case the third stage is the same as Section 2.1.

At the second stage, let ϕ_1 denote the profit of the ancillary services of facility 1, which is given by $\phi_1 = v_1 x_1 - \delta v_1^2/2$. From the first-order conditions we have,

$$\frac{\partial \phi_1}{\partial v_1} = \frac{a(1+c)-v_2}{1+3c+c^2} - \delta v_1 = 0. \tag{20}$$

Firm 2's first-order condition is the same as (12). Then, solving (12) and (20) we obtain the levels of investment in this lump-sum outsourcing regime, v_1^L and v_2^L, as follows:

$$\begin{aligned} v_1^L &= \frac{a(1+c)[-2-c+\delta(1+3c+c^2)]}{-\delta(1+c)^2(2+c)+\delta^2(1+3c+c^2)^2}, \\ v_2^L &= \frac{ac(1+c)(2+c)}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}. \end{aligned} \tag{21}$$

One can see from (12) that v_2 does not depend on v_1. Hence, v_2^L is equal to v_2^M. Furthermore, one can see from (7), (8) and (9) that neither x_1, x_2, π_1 nor π_2 depends on v_1. Therefore we have the amounts produced, x_1^L and x_2^L, as below:

$$x_1^L = x_1^M = \frac{a(1+c)[-2-c+\delta(1+3c+c^2)]}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}, \tag{22}$$

$$x_2^L = x_2^M = \frac{ac\delta(1+3c+c^2)}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}. \tag{23}$$

The only difference from the government's operation is that v_1 is positive, and hence, facility 1's overall profit, Π_1, and the social welfare are larger. Substituting (8), (9), (21)–(23) into (10) we have the social welfare, SW_L, as follows:

$$SW^L = \frac{\Theta}{2\delta[(1+c)^2(2+c)-\delta(1+3c+c^2)]^2},$$

where,

$$\begin{aligned} \Theta \equiv a^2[&(1+c)^2(2+c)^2 - c\delta(1+c)^2(2+c)(3+c) \\ &\delta^2(1+c)(3+19c+43c^2+37c^3+14c^4+2c^5) \\ &\delta^3(1+3c+c^2)^2(1+5c+8c^2+2c^3)]. \end{aligned}$$

If there is no technology to reduce the marginal cost of the ancillary services, the equilib-

rium is the same as that of the government's operation. Therefore, the social welfare, SW^{L0}, is equal to SW^{M0}.

2.3 Privatization

Now let us consider the case where a concession-based PPP is introduced. Firm 1 acts as a private firm and its profit is described as below:

$$\pi_1 = px_1 - \frac{c}{2}x_1^2 + v_1 x_1 = (a + v_2 - x_1 - x_2)x_1 - \frac{c}{2}x_1^2. \tag{25}$$

At the third stage, the two firms maximize their profits (25) and (3). From the first-order conditions we have,

$$\frac{\partial \pi_i}{\partial x_i} = a + v_i - (2+c)x_i - x_j = 0, \quad i, j = 1, 2 \quad i \neq j. \tag{26}$$

Solving (26) for x_1 and x_2 yields,

$$x_i = \frac{a(1+c) + (2+c)v_i - v_j}{(1+c)(3+c)}, \quad i, j = 1, 2, \quad i \neq j. \tag{27}$$

Substituting (27) back into (25) and (3) we have,

$$\pi_i = \frac{(2+c)[a(1+c) + (2+c)v_i - v_j]^2}{2(1+c)^2(3+c)^2}, \quad i, j = 1, 2, \quad i \neq j. \tag{28}$$

At the second stage, the two firms choose the levels of investment for the ancillary services. The overall profit for firm 1, Π_1, is now given by $\Pi_1 = \pi_1 - \delta v_1^2/2$, and the social welfare is described as below:

$$SW = CS + \Pi_1 + \Pi_2. \tag{29}$$

As for firms, from the first-order conditions we have,

$$\frac{\partial \Pi_i}{\partial v_i} = \frac{(2+c)^2[a(1+c) + (2+c)v_i - v_j] - (3 + 4c + c^2)^2 v_i \delta}{(1+c)^2(3+c)^2} = 0, \quad i, j = 1, 2 \quad i \neq j \tag{30}$$

Solving (30), the levels of investments in this pure duopoly, v_1^P and v_2^P, are as follows:

$$v_i^P = \frac{a(2+c)^2}{-(2+c)^2 + \delta(1+c)(3+c)^2}, \quad i = 1, 2. \tag{31}$$

In order for v_1^P and v_2^P to be positive the following condition is necessary:

$$\delta > \frac{(2+c)^2}{(1+c)(3+c)^2}, \tag{32}$$

which we assume in what follows. Substituting (31) into (27) we have the amounts produced, x_1^P

and x_2^P, as follows:

$$x_i^P = \frac{a\delta(1+c)(3+c)}{-(2+c)^2+\delta(1+c)(3+c)^2}, \quad i=1,2. \tag{33}$$

Therefore, substituting (31) and (33) into (29) yields the social welfare, SW^P, as follows:

$$SW^P = \frac{a^2[-(2+c)^4\delta+(4+c)(3+4c+c^2)^2\delta^2]}{(2+c)^4-2(1+c)(6+5c+c^2)^2\delta+(1+c)^2(3+c)^4\delta^2}. \tag{34}$$

Suppose again that there is no technology for cost-reducing investment for the ancillary services. That is, the market is an ordinary pure duopoly. Substituting $v_1=0$ and $v_2=0$ into (27) yields the amounts produced, x_1^{P0} and x_2^{P0}, as follows:

$$x_i^{P0} = \frac{a}{3+c}, \quad i=1,2. \tag{35}$$

Then we have the social welfare, SW^{P0}, as below:

$$SW^{P0} = \frac{a^2(4+c)}{(3+c)^2}. \tag{36}$$

3. Comparison

In this section we compare the amounts produced and the social welfares under the three regimes, government's operation, lump-sum outsourcing, and privatization.

3.1 No investments

Let us first consider the case where there is no technology that allows firms or contractors to make the cost-reducing investment.

(1) Production

From (18) the difference in production by the public and private firms in the mixed duopoly is as follows:

$$x_1^{M0}-x_2^{M0} = \frac{a}{1+3c+c^2} > 0. \tag{37}$$

That is, the public firm produces more than the private firm. This result also holds in the case of lump-sum outsourcing.

From (18) and (33), the difference in production by firm 1 before and after privatization is as below:

$$x_1^{P0} - x_1^{M0} = -\frac{a(2+c)}{(3+c)(1+3c+c^2)} < 0. \tag{38}$$

Hence, the privatized firm produces less than it did before privatization. Similarly, the difference in production by firm 2 before and after privatization of firm 1 is as follows:

$$x_2^{P0} - x_2^{M0} = \frac{a}{(3+c)(1+3c+c^2)} > 0. \tag{39}$$

That is, firm 2 produces more than it did before privatization of firm 1. The difference in total production before and after privatization of firm 1 is as below:

$$(x_1^{P0} + x_2^{P0}) - (x_1^{M0} + x_2^{M0}) = \frac{-a(1+c)}{(3+c)(1+3c+c^2)} < 0. \tag{40}$$

Therefore, the total production always decreases after privatization of firm 1, and hence, so does the consumer surplus. The model here is an ordinary duopoly and the results above are well known.

(2) Social welfare

Let us turn to the social welfare. Using (19) and (36), subtracting the social welfare under government's operation, SW^{M0} $(= SW^{L0})$, from that under privatization, SW^{P0}, we have,

$$SW^{P0} - SW^{M0} = -\frac{1+c+3c^2+c^3}{2(3+c)^2(1+3c+c^2)^2} < 0. \tag{41}$$

That is, the social welfare is higher under government's operation than under privatization, and privatization is not desirable. This is a standard result with mixed duopoly which one can see in De Fraja and Delbono (1989) and others.

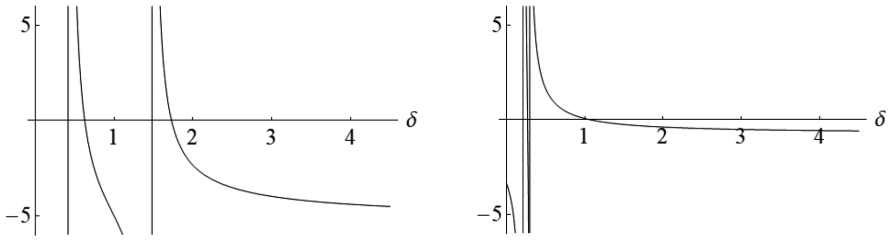
3.2 When investments are possible

Now we take into account the effects of cost-reducing investments, and compare the amounts produced and the social welfares.

(1) Production

Does the public firm in mixed duopoly produce more than the private firm as in Subsection 3.1? From (15) and (16) the difference in production by the two firms are as follows:

$$x_1^M - x_2^M = \frac{a[-(1+c)(2+c)+\delta(1+3c+c^2)]}{-(1+c)^2(2+c)+\delta(1+3c+c^2)^2}, \tag{42}$$



(i) $a = 10, \quad c = 0.1$ (ii) $a = 10, \quad c = 2$

Figure 1: Graphs of $x_1^P - x_1^M$

where the denominator is positive from the condition (14). Therefore, in order for $x_1^M > x_2^M$ to hold, the following condition is necessary:

$$\delta > \frac{(1+c)(2+c)}{1+3c+c^2}. \tag{43}$$

Which condition is more binding, (14) or (43)? Let $\bar{\delta}$ denote δ where (14) holds as an equality and $\hat{\delta}$ denote δ where $x_1^M = x_2^M$. Then we have,

$$\bar{\delta} - \hat{\delta} = -\frac{c(1+c)(2+c)^2}{(1+3c+2c^2)^2} < 0. \tag{44}$$

That is, $\bar{\delta} < \hat{\delta}$ and hence the condition (43) is more binding. This means that if δ satisfies (14) but is small enough (the investment cost is low enough) the private firm produces more than the public firm ($x_1^M < x_2^M$). If δ is large enough and satisfies (43) the public firm produces more than the private firm.[2)]

Next, using (15) and (33), we would like to see if the privatized firm produces more than it did before privatization. Unfortunately, however, $x_1^P - x_1^M$ is too complex and one cannot tell its sign analytically. Therefore, we set the values of parameters a and c, and plot $x_1^P - x_1^M$ as a function of δ.

The panel (i) of Figure 1 shows the case where $a = 10$ and $c = 0.1$, while the panel (ii) shows the case where $a = 10$ and $c = 2$. The part where δ is small and the graph is unstable corresponds to the range where δ does not satisfy the condition (14), and hence one can ignore it.[3)] The graphs are downward sloping where $\delta > \bar{\delta}$. One can say that $x_1^P > x_1^M$ where δ is small enough, and $x_1^P < x_1^M$ otherwise.[4)] That is, the privatized firm produces more than it did before privatization if the investment cost is low enough.

(2) Social welfare

Next let us compare the social welfare when firm 1 is under government's operation, SW^M, with that under lump-sum outsourcing, SW^L, and privatization, SW^P. We again set parameters a and c, and plot these welfares as functions of δ. Figure 2 shows the case where $a = 10$ and $c = 2$.

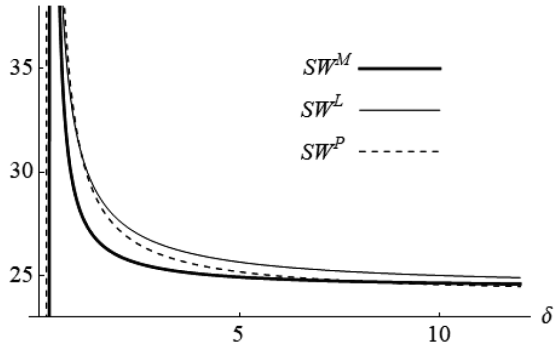


Figure 2: Graphs of SW^M, SW^L, and SW^P

One can see from Figure 2 that SW^P is the highest where δ is small enough. That is, the society is better off under privatization if the investment cost is low enough. SW^P is, however, the lowest where δ is large enough because the cost reduction from the investment becomes very small and the advantage from maximizing the government's objective function becomes dominant. On the other hand, SW^L is the highest where δ is not small. In that case, lump-sum outsourcing is the most desirable.

Looking back at the history of privatization, methods such as lump-sum outsourcing were introduced first, followed by privatization (or concession-based PPP). This can be attributed to a decline in investment costs due to technological innovations, in addition to the emergence of financial methods such as project financing.

(3) Numerical examples

Now let us see the numerical examples of the regimes mentioned above. Table 1 shows the case where $a = 10$, $c = 2$, and $\delta = 0.5$. In this case the condition (43) is not satisfied and the private firm (firm 2) produces more than the public firm (firm 1) in mixed duopoly. On the other hand, firm 1's production increases after privatization, or in pure duopoly, because δ is small enough.

Table 2 shows the case where $a = 10$, $c = 2$, and $\delta = 2$. The investment cost δ satisfies the condition (43), and hence the public firm produces more than the private firm in mixed duopoly. On the other hand, firm 1's production decreases after privatization because δ is relatively large.

In Table 1, the total production increases after privatization of firm 1, and hence, so does the consumer surplus. In table 2, however, the total production decreases after privatization and so does the consumer surplus. The total production and the consumer surplus can increase after privatization if δ is low enough.

The social welfare under privatization is the highest in Table 1 because δ is small enough. In Table 2, however, the social welfare under lump-sum outsourcing is higher because δ is not small.

Table 1: Values of variables where $a = 10$, $c = 2$, *and* $\delta = 0.5$

	x_1	x_2	p	v_1	v_2	CS	Π_1	Π_2	SW
Government's operation	1.84	4.49	3.67	0.00	9.80	20.01	3.37	16.33	39.71
Lump-sum outsourcing	1.84	4.49	3.67	0.00	9.80	20.01	6.74	16.33	43.09
Privatization	3.49	3.49	3.02	7.44	7.44	24.34	10.49	10.49	45.32

Table 2: Values of variables where $a = 10$, $c = 2$, *and* $\delta = 2$

	x_1	x_2	p	v_1	v_2	CS	Π_1	Π_2	SW
Government's operation	2.62	2.14	5.24	0.00	1.17	11.32	6.87	7.77	25.95
Lump-sum outsourcing	2.62	2.14	5.24	1.31	1.17	11.32	8.59	7.77	27.67
Privatization	2.24	2.24	5.52	1.19	1.19	10.02	8.60	8.60	27.22

4. Conclusion

While privatization has taken various forms in these decades, the theory of mixed oligopoly does not fully reflect it. In addition, in standard models of mixed duopoly, the public firm produces more than the private firm in order to maximize the social welfare. It decreases production after privatization in order to maximize profit, and the social welfare is reduced. They are, however, not necessarily true in reality. In the present paper we introduced *ex ante* investments for cost reduction and the choice of bundling or unbundling of tasks into the model of mixed duopoly.

We showed that if the investment cost is low enough the private firm produces more than the public firm in mixed duopoly, and the privatized firm in pure duopoly produces more than it did before privatization. The social welfare is higher under privatization than under other regimes if the investment cost is low enough.

In the present paper we assumed a homogeneous good. We could extend the model to examine the case where the good is differentiated. While we considered a cost-reducing investment, we could examine the effects of a quality-enhancing investment. These are left for future research.

Notes

1) The sum of the profits of firm 1 and the ancillary services minus investment cost, or the overall profit of facility 1, Π_1, would be $\Pi_1 = \pi_1 + v_1 x_1 - \delta v_1^2/2$.

2) Numerical examples are $\hat{\delta} \approx 1.76$ if $c = 0.1$, while $\hat{\delta} \approx 1.09$ if $c = 2$.

3) $\overline{\delta} \approx 1.48$ in panel (i) and $\overline{\delta} \approx 0.30$ in panel (ii). The condition (32) is weaker than (14) and hence is not binding.

4) Let $\tilde{\delta}$ denote δ where $x_1^P = x_1^M$. Then, $\tilde{\delta} \approx 1.72$ if $c = 0.1$, while $\tilde{\delta} \approx 1.07$ if $c = 2$.

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（要旨）

近年，民営化は各種の官民協働（PPP）など様々な形態をとるが，混合寡占の理論はそれを十分に反映していない。混合複占の標準的なモデルでは，公企業は社会厚生を最大化するため民間企業よりも多く生産する。それが民営化されると利潤最大化のため生産量を減らし，社会厚生は減少する。しかし，現実には必ずしもそのようにならず，民営化後に生産量を増やし，社会厚生が上昇する場合もあると考えられる。本論文では付随的業務の費用削減のための事前投資と，業務のバンドリング・アンバンドリングの選択を混合複占のモデルに導入し，以下のことを示す。投資費用が十分に低ければ，混合複占において民間企業は公企業よりも多く生産する。その公企業は民営化されると，民営化前よりも生産量を増やす。また社会厚生は民営化の場合に最も高くなる。投資費用が一定水準以上であれば，付随的業務を一括外注した場合に社会厚生が民営化や公営の場合よりも高くなる。