【Articles】

Vertical and Horizontal Competition between Public Firms and Privatization

公企業間の垂直的・水平的競争および民営化

OSHIMA Kosuke
大　島　考　介

Contents

(Abstract)

The literature of studies on privatization using models of mixed oligopoly and public duopoly has a long history. The issues of these studies include partial privatization, multiple countries or regions, differentiated goods, centralization and decentralization, and others. Competitions in those models are basically horizontal ones, such as competition between local public firms and between public and private firms. However, little attention has been paid to the research on vertical competition in this area such as that between state-owned and local public firms. The present paper first sets up a simple two-city model of horizontal competition and analyzes the equilibrium. We should note that the privatization in one city can change the objective function of the other city's public firm if its residents own the stock of the privatized firm. Then we modify the model into that of a vertical competition and investigate the results. We find that vertical and horizontal competitions between public firms can achieve the same equilibrium although the reaction functions of the firms are different. If the city-owned firm is privatized a different equilibrium is realized, but the welfare levels of the cities remain the same. The model of the present paper can be used as a benchmark and may be extended in various ways. For example, one may assume that the cities are asymmetric in population or others. Alternatively, one can consider differentiated goods, partial privatization, or public-private partnerships.

『商学集志』第 92 巻第 1 号（'22.6）

1. Introduction

The literature of studies on privatization using models of mixed oligopoly and public duopoly has a long history. Mixed oligopoly is a market where a small number of firms, public and private, compete with each other. Public duopoly is a market with two public firms owned by different countries or local governments. While private firms maximize profits, a public firm acts in order to maximize (national or local) social welfare. If it is privatized it acts as a private firm and hence affects the social welfare. Therefore it is important to consider how the government should handle those public firms.

Since De Fraja and Delbono's (1989) seminal paper many researchers focused on various issues in this area. Matsumura (1998) took into account partial privatization in mixed duopoly model and showed that partial privatization can be more desirable than full nationalization or full privatization. Matsumura and Kanda (2005) considered the case where firms can enter the market freely and they showed that the government should fully own the public firm. Fujiwara (2007) introduced product differentiation into the model and showed that partial privatization is optimal except in extreme cases.[1)] Researchers such as Bárcena-Ruiz and Garzón (2005) and Dadpay and Heywood (2006) considered two-country models and investigated strategic interactions between two countries in deciding whether to privatize their public firms. Han and Ogawa (2008), allowing for partial privatization, examined the impact of the market integration of two countries and showed that the governments are less eager to privatize in the international mixed oligopoly market than in a single-country framework. Mantin (2012) and Matsumura and Matsushima (2012) investigated public duopoly models where each country owns an airport and decides whether to privatize it. In their models the strategic interaction between the airports is strategic substitution. They showed that the two countries privatize their airports despite higher welfare levels under non-privatization by both countries. That is, the governments face the prisoner's dilemma. Oshima (2018) considered a differentiated mixed duopoly with partial privatization in a two-city model where a public firm operate in one city and a private firm in the other, and compared decentralized and centralized solutions. The result is that in most cases the privatization level is higher under centralization (the state owns the public firm) than under decentralization (the city owns the public firm). In particular, under decentralization, the public firm is fully owned by the city government if the substitutability of differentiated goods is relatively high.

Competitions in those models mentioned above are basically horizontal ones. They are, for example, competitions between local public firms and between public and private firms. However, little attention has been paid to vertical competition in this area. Here the phrase vertical competition refers to a competition between firms owned by different levels of governments, such as national and local governments. An example of vertical competition would be airports. One can imagine state-owned, local government-owned, and private airports competing in a certain region. One may also consider prefectural and municipal museums operating in

a certain city. How the firms interact with each other can affect the efficiency of the market. We should also note that the privatization in one city changes the objective function of public firm in the other city if its residents own the stock of the privatized firm. In the present paper we compare horizontal and vertical competitions using a simple two-city model, which can be a benchmark for further research.

The rest of the paper is organized as follows. In section 2 we set up a model commonly used for horizontal and vertical competitions. In section 3 we derive the equilibria of horizontal competition and evaluate the social welfare levels. Section 4 provides the equilibria of vertical competition and a comparison with horizontal competition. Section 5 concludes.

2. The model

Suppose a country that consists of two cities, city 1 and city 2 with the same population. The residents are homogeneous. We standardize the total population to unity, and hence the population of each city is 1/2. Each city has a public firm. City 1's firm (firm 1) and city 2's firm (firm 2) produce a homogeneous good. In Mantin (2012) and Matsumura and Matsushima (2012) the two airports are located in two different countries, respectively, and airlines connect the two airports. Therefore, they are complements. In the present paper, however, the goods produced by the two firms are (perfect) substitutes. Taking airports as an example, the two airports are located in neighboring cities, respectively, and airlines connect each of them with foreign airports. They are competing with each other and hence are substitutes.

The profit of a public firm belongs to the city which owns the firm. If the firm is privatized its stock is widely sold to residents around the country. Let us assume that the stock is evenly owned by the residents of two cities. Therefore the profit of the privatized firm is distributed in half to the two cities.

The inverse demand function of the good is expressed as below:

$$p = a - (q_1 + q_2), \tag{1}$$

where p, q_1 and q_2 denote the price and quantities of the good produced by firms 1 and 2, respectively. Parameter a shows the size of the market. From (1) the consumer surplus of this market is as follows:

$$CS = \frac{(q_1 + q_2)^2}{2}. \tag{2}$$

Now we consider the producers. Suppose that the firms face the same technology and the cost function $C_i = cq_i + f$, $i = 1, 2$, where c is the marginal cost, $c < a$, and f is the fixed cost. As shown in De Fraja and Delbono (1989) and others the increasing marginal cost works in favor of privatization. Hence we assume that the marginal cost is constant, which is a usual assumption in the literature.[2) 3)] Then the profit of firm i is as follows:

$$\pi_i = (p-c)q_i - f, \quad i = 1,2. \tag{3}$$

The social welfare of the country, W, is defined as the sum of consumer surplus and profits of the two firms:

$$W \equiv CS + \pi_1 + \pi_2 . \tag{4}$$

Because the two cities are symmetric in consumption, consumer surplus for each city is $CS/2$. Therefore the welfares of the two cities, W^1, W^2 are defined as follows;

$$W^i \equiv CS/2 + \pi_i , \quad i = 1,2, \tag{5}$$

no matter whether firms are privatized or not. If firm i is privatized $\pi_i/2$ belongs to city j ($j = 1,2$, $j \neq i$). Assuming perfect foresight, however, city i receives the same amount from residents in city j when it sells the firm's stock.

We consider a two-stage game. In the first stage the cities 1 and 2 choose whether to privatize their public firms. In the second stage the firms determine the quantities supplied simultaneously (that is, we consider a Cournot competition). Then we solve the game by backward induction.

In the second stage, if firm 1 is privatized, it maximizes its profit π_1. From (1) and (3) we have

$$\max \pi_1 = (a - c - q_1 - q_2)q_1 - f .$$

From the first-order condition we have firm 1's reaction function as follows:

$$q_1 = \frac{a - c - q_2}{2}. \tag{6}$$

Similarly, if firm 2 is also privatized, one can obtain firm 2's reaction function as below:

$$q_2 = \frac{a - c - q_1}{2}. \tag{7}$$

On the other hand, if both firms are city-owned firm 1 maximizes W^1. Therefore, from (1) – (3) and (5) we have

$$\max W^1 = \frac{(q_1 + q_2)^2}{4} + (a - c - q_1 - q_2)q_1 - f .$$

If, however, only firm 2 is privatized, firm 1 maximizes $W^1 + \pi_2/2$. That is, while the welfare of city i is expressed as (5) the incentive of the city is affected if firm j is privatized and the residents of city i own half of the stock of firm j. Therefore we have

$$\max W^1 + \frac{\pi_2}{2} = \frac{(q_1 + q_2)^2}{4} + (a - c - q_1 - q_2)\left(q_1 + \frac{q_2}{2}\right) - \frac{3f}{2}.$$

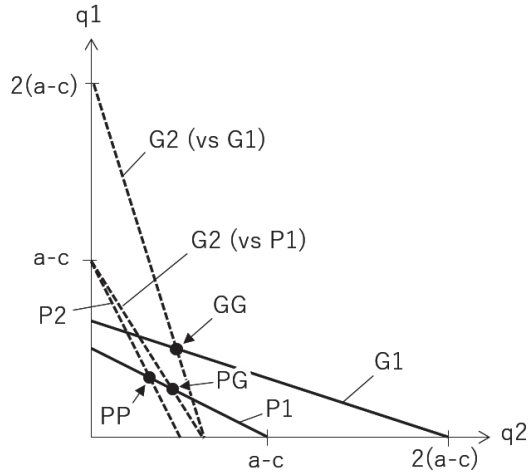

Figure 1: Horizontal competition

From the first-order condition we obtain public firm 1's reaction function as follows:

$$q_1 = \begin{cases} \dfrac{2a - 2c - q_2}{3} & \text{if firm 2 is city-owned,} \\ \dfrac{2(a - c - q_2)}{3} & \text{if firm 2 is privatized.} \end{cases} \tag{8}$$

Similarly, we have public firm 2's reaction function as below:

$$q_2 = \begin{cases} \dfrac{2a - 2c - q_1}{3} & \text{if firm 1 is city-owned,} \\ \dfrac{2(a - c - q_1)}{3} & \text{if firm 1 is privatized.} \end{cases} \tag{9}$$

The intuition is that the public firm i produces less if firm j is privatized than if it is city-owned, and let firm j produce more. This is because city i takes into account the interests of its residents as shareholders of firm j.

3. Horizontal competition

Let PP denote competition between private firms 1 and 2. Similarly, PG denotes competition between private firm 1 and public firm 2 while GG denotes that between public firms 1 and 2. From (6) – (9) one can draw reaction curbs of the two firms under GG, PG and PP as in Figure 1.[4] P1 and P2 represent the reaction curves of private firms 1 and 2, while G1 and G2 those of public firms 1 and 2. The points show the equilibria of the three types of competition.

Now let us see the equilibrium. Under PP, from (6) and (7) we have the equilibrium production levels as follows:

Table 1: Payoff matrix of horizontal competition

		firm 2	
		P	G
firm 1	P	$\frac{2}{9}(a-c)^2-f,\quad \frac{2}{9}(a-c)^2-f$	$\frac{13}{64}(a-c)^2-f,\quad \frac{17}{64}(a-c)^2-f$
	G	$\frac{17}{64}(a-c)^2-f,\quad \frac{13}{64}(a-c)^2-f$	$\frac{1}{4}(a-c)^2-f,\quad \frac{1}{4}(a-c)^2-f$

$$q_1^{PP}=q_2^{PP}=\frac{a-c}{3}, \tag{10}$$

where q_1^{PP} and q_2^{PP} denote equilibrium production levels of firms 1 and 2 under PP. Substituting (10) into (4) and (5) we have the levels of social welfare W^{PP} and regional welfares W^{1PP} and W^{2PP} as follows:

$$W^{PP}=\frac{4}{9}(a-c)^2-2f,\qquad W^{1PP}=W^{2PP}=\frac{2}{9}(a-c)^2-f. \tag{11}$$

Under PG, from (6) and (9) we have

$$q_1^{PG}=\frac{a-c}{4},\qquad q_2^{PG}=\frac{a-c}{2}. \tag{12}$$

Substituting (12) into (4) and (5) yields

$$W^{PG}=\frac{15}{32}(a-c)^2-2f,\quad W^{1PG}=\frac{13}{64}(a-c)^2-f,\quad W^{2PG}=\frac{17}{64}(a-c)^2-f. \tag{13}$$

Under GG, from (8) and (9) we have

$$q_1^{GG}=q_2^{GG}=\frac{a-c}{2}. \tag{14}$$

Substituting (14) into (4) and (5) yields

$$W^{GG}=\frac{1}{2}(a-c)^2-2f,\qquad W^{1GG}=W^{2GG}=\frac{1}{4}(a-c)^2-f. \tag{15}$$

We can now compare the welfare levels under PP, PG, and GG. From (11), (13), and (15) we have the following inequality:

$$W^{PP}<W^{PG}<W^{GG}. \tag{16}$$

One can see that GG is the most preferable as a whole in this simple model. From (11), (13), and (15) we can also make the payoff matrix of horizontal competition as Table 1. Because public ownership is the dominant strategy for both cities the Nash equilibrium is (G, G). Under PG and GP the city which does not privatize its public firm is better off than under GG, but it is not feasible. The assumption of public firm's objective function when the other city's firm is privatized, which was mentioned in the last section, affects the payoffs under PG and GP. The Nash equilibrium is unaffected, though.[5)]

4. Vertical competition

Suppose now that the national government owns one of public firms. Without loss of generality, we assume that the national government owns firm 1 and city 2 owns firm 2. That is, the vertical competition arises where the firms of national and local governments compete in the market. The state-owned firm 1 maximizes the social welfare of the country, W. Therefore, from (2) – (4) we have

$$\max W = \frac{(q_1 + q_2)^2}{2} + (a - c - q_1 - q_2)(q_1 + q_2) - 2f.$$

From the first-order condition we obtain firm 1's reaction function as follows:

$$q_1 = a - c - q_2. \tag{17}$$

The objective function of state-owned firm 1, and hence its reaction function, is not affected no matter whether firm 2 is privatized or not. This is because firm 1's objective is to maximize the country's social welfare, or the sum of total consumer surplus and total profits, which is not affected by the privatization of firm 2.

Let NG denote competition between state-owned firm 1 and city-owned firm 2. Similarly, NP denotes competition between state-owned firm 1 and private firm 2. From (7), (9), and (17) one can draw reaction curbs of the two firms, firm 1 and firm 2, under NG and NP as in Figure 2.[6)]

Now let us see the equilibrium. Under NG, from (9) and (17) we have the equilibrium production levels as follows:

$$q_1^{NG} = q_2^{NG} = \frac{a - c}{2}. \tag{18}$$

Substituting (18) into (4) and (5) we have the levels of social welfare and regional welfares as follows:

$$W^{NG} = \frac{(a - c)^2}{2} - 2f, \quad W^{1NG} = W^{2NG} = \frac{(a - c)^2}{4} - f. \tag{19}$$

From (14), (15), (18), and (19) one can see that NG achieves the same equilibrium as GG although firm 1's reaction curb is different from that of GG (see Figure 2).

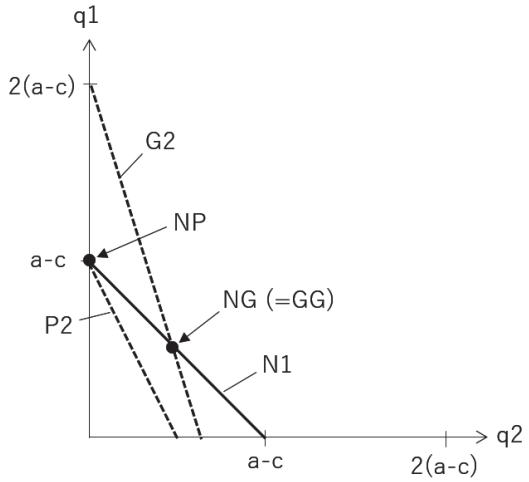

Figure 2: Vertical competition

Under NP, from (7) and (17) we have the equilibrium production levels as follows:

$$q_1^{NP} = a - c, \quad q_2^{NP} = 0. \tag{20}$$

Substituting (20) into (4) and (5) yields

$$W^{NP} = \frac{(a-c)^2}{2} - 2f, \quad W^{1NP} = W^{2NP} = \frac{(a-c)^2}{4} - f. \tag{21}$$

Therefore, from (15), (19), and (21) one can see that NP achieves the same welfare level as GG and NG.

From (11), (13), (19), and (21) we have the payoff matrix of vertical competition as Table 2. There exist two Nash equilibria, (N, G) and (N, P) in this game. Firms 1 and 2 are, however, originally owned by national and city governments, respectively. That is, (N, G) is the status quo. In addition, as one can see from (20), the private firm 2 gets crowded out by state-owned firm 1 in the equilibrium (N, P). Therefore one can predict that city 2 will not privatize firm 2 if firm 1 is state-owned.

As far as the welfare level is concerned, the equilibria (N, G) and (N, P) achieve the same welfare level and are Pareto efficient. This is because the slope of the reaction function of the state-owned firm is –1 and firms produce and sell goods at the marginal cost, c. That is, there is a trade-off between the amounts of goods produced by the two firms. This result holds even if private firm 2's reaction function is different (e.g., $q_2 = 2(a - c)/3 - q_1/2$) and both firms produce positive amounts of goods under NP.

Table 2: Payoff matrix of vertical competition

		firm 2	
		P	G
firm 1	P	$\frac{2}{9}(a-c)^2-f,\quad \frac{2}{9}(a-c)^2-f$	$\frac{13}{64}(a-c)^2-f,\quad \frac{17}{64}(a-c)^2-f$
	N	$\frac{1}{4}(a-c)^2-f,\quad \frac{1}{4}(a-c)^2-f$	$\frac{1}{4}(a-c)^2-f,\quad \frac{1}{4}(a-c)^2-f$

5. Conclusion

The literature of studies on mixed oligopoly and public duopoly has a long history and various issues have been investigated such as partial privatization, multiple countries or regions, differentiated goods, centralization and decentralization, and others. While these studies basically investigated horizontal competitions, little attention has been paid to the research on vertical competitions in this area.

In the present paper we first considered horizontal competition using a simple two-city model. Assuming that city i's residents own half of firm j's stock if it is privatized, the objective function of city i's public firm is affected so that it takes into account the interests of city i's residents as shareholders of private firm j. Then the dominant strategy for both cities is public ownership and the Nash equilibrium is (G, G). Next we supposed that the national government owns firm 1 and city 2 owns firm 2, and investigated the vertical competition. As the objective of state-owned firm 1 is to maximize the social welfare of the country as a whole, its objective function, and hence, its reaction function, is not affected no matter whether firm 2 is privatized or not. As a result, although the reaction function of firm 1 under NG (vertical competition) is different from that under GG (horizontal competition), NG and GG achieve the same equilibrium. Because the slope of firm 1's reaction curb is –1 NP achieves the same welfare level as under NG and GG in equilibrium. In that case, however, the private firm 2 gets crowded out by state-owned firm 1 and hence city 2 will not privatize firm 2.

We found that in our simple model the governments do not sell their firms. This is true in many cases of "privatization." Recently, for example, the operation of many airports, state-run and local government-run, were left to private firms, while the governments still own those airports. In order to take into account complex realities, however, we need to consider various extensions to the model.

For example, one may assume that the cities are asymmetric in population, productivity, or others. It may be meaningful to suppose that the goods which the firms produce are differentiated. One can also introduce partial privatization into the model, where the public firm maximizes the weighted average of social welfare and firm's profit. Recently, however, participation of private firms in public works projects often takes the form of public-private

partnerships, as in airports mentioned above. Therefore it may be worth introducing such an approach into the model. These are left for future research.

Notes

1) See also Anderson et al. (1997), Saha (2009) and Choi (2012) for product differentiation.
2) We also assume that firms cannot discriminate local consumers from consumers living in the other city.
3) In the present model we do not assume transportation costs to avoid complexities.
4) GP is symmetrical to PG. Therefore, in order to avoid complication we omit GP in Figure 1.
5) If the public firm 1 disregarded the profit of private firm 2, the welfare levels of cities 1 and 2 under GP would be $7(a-c)^2/25-f$ and $(a-c)^2/5-f$ instead of $17(a-c)^2/64-f$ and $13(a-c)^2/64-f$. Under PG, the welfare levels of cities 1 and 2 would be $(a-c)^2/5-f$ and $7(a-c)^2/25-f$. Because $2/9 < 7/25$ and $1/5 < 1/4$, (G, G) is the Nash equilibrium in this case, too. Since $17/64 < 7/25$ and $13/64 > 1/5$, however, the actual Nash equilibrium is closer to an opposite solution (P, P).
6) One should read "city-owned" in (8) and (9) as "state-owned" here.

References

Anderson, S. P., A. de Palma and J.-F. Thisse (1997) "Privatization and Efficiency in a Differentiated Industry," *European Economic Review* Vol. 41, No. 9, pp.1635-1654.

Bárcena-Ruiz, J. C. and M. B. Garzón (2005) "International Trade and Strategic Privatization," *Review of Development Economics* Vol. 9, No. 4, pp.502-513.

Choi, K. (2012) "Cournot and Bertrand competition with asymmetric costs in a mixed duopoly revisited," MPRA Paper, No. 37004

Dadpay, A. and J. S. Heywood (2006) "Mixed Oligopoly in a Single International Market," *Australian Economic Papers* Vol. 45, No. 4, pp.269-280.

De Fraja, G. and F. Delbono (1989) "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers* Vol. 41, No. 2, pp.302-311.

Fujiwara, K. (2007) "Partial Privatization in a Differentiated Mixed Oligopoly," *Journal of Economics* Vol. 92, No. 1, pp.51-65.

Han, L. and H. Ogawa (2008) "Economic Integration and Strategic Privatization in an International Mixed Oligopoly," *FinanzArchiv* Vol. 64, No. 3, pp.352-363.

Mantin, B. (2012) "Airport complementarity: Private vs. government ownership and welfare gravitation," *Transportation Research Part B* Vol. 46, pp.381-388.

Matsumura, T. (1998) "Partial Privatization in Mixed Duopoly," *Journal of Public Economics* Vol. 70, No. 3, pp.473-483.

Matsumura, T. and O. Kanda (2005) "Mixed Oligopoly at Free Entry Markets," *Journal of Economics* Vol. 84, No. 1, pp.27-48.

Matsumura, T. and N. Matsushima (2012) "Airport Privatization and International Competition," *The Japanese Economic Review* Vol. 63, No. 4, pp.431-450.

Oshima, K. (2018) “Differentiated mixed duopoly and decentralization in a two-city model,” *Papers in Regional Science* Vol. 97, No. 4, pp.1425-1440.

Saha, B. (2009) “Mixed ownership in a mixed duopoly with differentiated products,” *Journal of Economics* Vol. 98, No. 1, pp.25-43.

（要旨）

混合寡占と公共複占のモデルを用いた民営化に関する研究には長い歴史がある。これらの研究の課題には，部分民営化，複数の国または地域，差別化された財，中央集権と地方分権などが含まれる。これらのモデルにおける競争は，地方公企業間の競争や公企業と民間企業間の競争など，基本的には水平的競争である。しかしこの分野では，国営企業と地方公企業間の競争のような垂直的競争に関する研究にはほとんど注意が払われてこなかった。本論文は，最初に水平的競争の単純な 2 都市モデルを構築し，均衡を分析する。ある都市での民営化は，他方の都市の住民が民営化された企業の株を所有している場合，他方の都市の公企業の目的関数を変えることに注意する必要がある。次に，モデルを垂直的競争に変更して結果を分析する。その結果，公企業の反応関数が異なるのにもかかわらず，公企業間の垂直的競争と水平的競争は同じ均衡を実現することが分かる。このとき都市所有の企業が民営化されても社会厚生水準は同じになるが，均衡は異なったものになる。本論文のモデルをベンチマークとして，様々な拡張を考えることができる。例えば，都市の人口等が非対称であると仮定してもよい。あるいは，差別化された財，部分民営化，または官民協働などを検討することができる。